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BIMODULAR THIN-WALLED BEAMS: A VARIATIONAL MODEL BASED ON A DUAL CONSTRAINED APPROACH

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ABSTRACT

Thin-walled composite elements made of fiber-reinforced polymer (FRP) are being increasingly used as primary and secondary members in civil and mechanical engineering structures. The main reason why these elements are often preferred to those ones constituted by more conventional materials, such as stainless steel or aluminium, is the good combination of the low weight, weak corrosion sensitivity, and structural strength they offer.

As it is experimentally confirmed, especially when soft matrices are involved, FRP materials can exhibit a different behaviour when loaded in tension or in compression. In detail, they may exhibit nonlinear behaviour characterized by a linear relationship between stress and strain both in tension and in compression, but with different elastic moduli. This behaviour is usually referred to as "bimodular" and in the case of fiber-reinforced composites it can be modelled through a strain-based unilateral constitutive approach, i.e. the constitutive behaviour depends on the sign of unit elongation in the fibers direction [1-3].

In this work the infinitesimal deformation theory is applied and a fiber-governed constitutive model is employed in the framework of the elastostatic problem of the thin-walled beam. Because of its specific geometry (the structural element beam has two dimension fairly smaller than the other one) the analysis of a composite beam is usually carried out by means of approximate one-dimensional models. The reduction of the three-dimensional elastostatic problem to a one-dimensional simplified one is performed by considering suitable assumptions on the strain and/or stress fields. A starting point for the development of composite beam theories in the case of thin-walled open section beams is represented by the Vlasov's theory [4]. It refers to isotropic homogeneous beams and consists essentially in subdividing in two parts the shear stresses flow generated by torsion: the primary flow of St. Venant's theory is associated to the so-called "pure torsion", and the secondary one is associated to the shear stresses induced by the non-uniform warping of the cross-section due to the primary flow. Although this theory has some limitations, it represents a general beam theory from which many authors have developed more refined approaches for the analysis of composite beams (e.g., [5-7]). Nevertheless, the rational deduction of these theories by

consistent assumptions on both stress and strain fields and their generalization to the case of bimodular anisotropic constitutive behaviour can be truly considered as an open task yet.

Starting from the three-dimensional elasticity theory and imposing frictionless dual internal constraints, as proposed by Bisegna and Sacco [8] and Maceri and Bisegna [9], a variational model for the analysis of bimodular thin-walled beams is deduced. Whereas the internal constraints approach proposed by Podio-Guidugli [10] is based on strain assumptions and on the concept of constrained material, the dual internal constraints method leaves unchanged the a-priori-given constitutive law (nonlinear in this case) and involves consistent and dual assumptions on both strain and stress fields. To this aim, the Hu-Washizu functional of the three-dimensional elasticity is modified according to the Lagrange multipliers theory and because of simultaneous presence of constraints on dual spaces, a non-standard application of the Lagrange theory is adopted. High-order effects concerning thickness and curvature of the cross-section centreline are taken into account and a refined expression of the torsional warping function for open cross-section beams is deduced.

It is worth observing that, due to the nonlinearity introduced by the unilateral constitutive behaviour, the proposed formulation involves a free-boundary problem concerning the determination of the neutral constitutive surface (zero-fiber-strain surface). Accordingly, a convergent iterative procedure is proposed.

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