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IDENTIFICATION OF THERMAL STRESSED STATE IN INHOMOGENEOUS THERMAL SENSITIVE CYLINDRICAL BODIES USING THE SURFACE DISPLACEMENTS

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ABSTRACT

The diagnosis and control of thermal and thermal stressed state in the operating parts of working heat-and-power equipment is often realized due to constructive, technological or methodical causes with incomplete data about their thermal loading. Thus the corresponding heat conductivity and thermoelasticity problems representing these states are underdetermined. If the original problem is supplied with additional information about the thermal processes (changes of temperature, heat flow) in several points of the construction part, then the identification of unknown thermal loading may be reduced to the solving of inverse ill-posed heat conductivity problems [1]. The diagnostic methods of thermal state in equipment using this approach are destructive. It is the main disadvantage of this method. Additionally there are sometimes impossible to represent all parameters of thermal loading after solving the inverse heat transfer problem [2]. If the original problem is supplied with information about thermo-mechanical parameters of the process (displacements, strains, stresses) in several points of the part, then the inverse thermoelasticity problem will be obtained [2, 3].

The identification problem of the time dependent temperature distribution on the boundary surface of the inhomogeneous in radial direction and thermal sensitive (thermal and mechanical characteristics of the material depend on the temperature) long hollow cylinder, its thermal and thermal stressed states with known deformations on the other surface is reduced to the inverse thermoelasticity problem. The last one is investigated.

Let us suppose that the cylinder is in plain strain state, the constringent loading exists on its surfaces, the initial temperature distribution and temperature changes of the external boundary surface are known. The temperature of the internal boundary surface, the thermal and thermal stressed states of the cylinder have been determined if additionally the time dependence of total strain $\varepsilon_r(1,\tau)+\varepsilon_\varphi(1,\tau)=\varphi(\tau)$ on the external boundary surface ($\varepsilon_r, \varepsilon_\varphi$ denote diagonal strain tensor components; τ is dimensionless time; $\varphi(\tau)$ is the known function) is known.

The formulated identification problem has been reduced to the solving of the inverse thermoelasticity one in the following way. The unknown boundary condition on the internal surface of the cylinder in the heat conductivity problem has been replaced with

the given condition on the strain on the external surface. The last one has been represented using results of the article [4] as the integral boundary condition for the temperature field $T(\rho,\tau)$:

$$\int_k^1 \eta \frac{\left(1+\nu(\eta,T)\right)\alpha(\eta,T)G(\eta,T)}{1-\nu(\eta,T)} T(\eta,\tau)d\eta + b\alpha(1,T)T(1,\tau) = \Psi(\tau), \qquad (1)$$

where k is the dimensionless internal radius of the cylinder; ν, α are the Poisson ratio and linear thermal expansion factor; G is a shear modulus; Ψ is the known function depending on given strains, on loading and temperature on the external surface. Thus the non-linear heat transfer problem with the integral boundary condition (1) has been obtained for the determining a temperature field.

The agreement condition at $\tau = 0$ under temperature distribution, given total strains and pressure inside and outside of the cylinder at the initial time moment has been obtained from the equality (1). The satisfying of the last one ensures the well posedness of the inverse thermoelasticity problem. The solution of this problem has been built using the finite difference method and the iteration procedure.

The proposed method for solving of the formulated identification problem has been approved for specific thermal and mechanical material characteristics. For this purpose the boundary thermal conditions has been set. A change of the total strains $\varepsilon_r + \varepsilon_\varphi$ on the external cylinder surface with the time has been determined as a solution of the direct thermoelasticity problem. The determined strains has been approximated with assigned accuracy (0.5%) by the cubic splines and has been used as known in the inverse problem. Using the proposed here approach the temperature on the internal surface as the solution of the corresponding inverse problem has been determined. The comparison of the obtained solution with the given temperature on the internal boundary surface in the direct problem makes clear, that the maximum deviation in the space of the continuous functions is not greater then 3%. Thus the satisfactory accuracy and stability of the inverse problem solution has been demonstrated. The thermal and thermal stressed states in the cylinder using the obtained thermal loading has been determined and investigated in [5].

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