

## A LOW-DISPERSIVE DYNAMIC FINITE DIFFERENCE SCHEME FOR LARGE EDDY SIMULATION.

D. Fauconnier\*, C. De Langhe and E. Dick

Ghent University, Department of Flow, Heat and Combustion Mechanics  
St. Pietersnieuwstraat 41, B-9000 Ghent, Belgium

e-mail: [dieter.fauconnier@Ugent.be](mailto:dieter.fauconnier@Ugent.be) - web page: <http://www.floheacom.ugent.be>

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### ABSTRACT

In the past decade, the necessity for numerical quality in Direct Numerical Simulations (DNS) and Large Eddy Simulations (LES) of turbulent flows, has been recognized by many authors among which Ghosal [1] and Chow *et al.* [2]. In a fully resolved DNS, the smallest resolved scales are located far into the dissipation range. The energy-content of those small scales is thus very small compared to those of the largest resolved scales. However, in LES, the smallest resolved scales are part of the inertial sub-range and thus contain still a significant amount of energy compared to the largest scales. Therefore, it might be reasonable to assume that numerical accuracy on the small scales is relatively more important for LES than for DNS. Moreover, some advanced subgrid modelling techniques such as the dynamic procedure or multiscale modelling strongly rely on the smallest resolved scales in LES, which make them even more important in terms of accuracy. Good numerical quality for an affordable LES is thus vital for accurate flow prediction as it directly influences resolved physics as well as subgrid physics.

Aside from aliasing errors, which should be prevented by eliminating scales beyond  $\kappa_c = \frac{2}{3}\kappa_{max}$ , finite difference errors are mainly responsible for the loss of numerical accuracy. Since it is highly desirable in LES, to maximize the ratio between the physical resolution and the grid resolution  $\frac{\kappa_c}{\kappa_{max}}$ , avoiding computational overhead, standard second order central schemes may not be sufficient. Ghosal [1] and Chow *et al.* [2] recommend a filter-to-grid ratio  $\frac{\kappa_c}{\kappa_{max}} = \frac{1}{4}$  when using second order central schemes. This could be prohibitively expensive for most computations. Therefore, one could apply higher order discretizations allowing larger filter-to-grid ratio's. However, acceptable dispersion errors up to  $\kappa_c = \frac{2}{3}\kappa_{max}$  require at least a standard tenth order central scheme, or compact Padé scheme, which leads again to increased complexity and/or computational costs.

In the present work, we develop a low-dispersive dynamic finite difference scheme for Large Eddy Simulation. The scheme, inspired by the work of Knaepen *et al.* [3], is constructed by combining Taylor expansions on 2 different grid resolutions which is reminiscent to Richardson Extrapolation. The technique has proved successful for obtaining higher accuracy in laminar flows in Fauconnier *et al.* [4]. Here, we refine the technique for Large Eddy Simulation. The resulting nonlinear scheme contains a dynamically obtained coefficient optimized according to the flow physics. The scheme leads to very high accuracy for the higher wavenumbers up to  $\frac{\kappa_c}{\kappa_{max}} = \frac{2}{3}$  while the accuracy on the lower wavenumbers remains at least second order. We also present a linearized version of this scheme leading to an

equivalent of the Dispersion-relation-preserving scheme of *Tam et al.* [5]. In contrast to the work of *Tam et al.* [5], we optimize the linearized scheme for smallest resolved scales close to  $\frac{\kappa_c}{\kappa_{max}} = \frac{2}{3}$  instead of  $\frac{\kappa_c}{\kappa_{max}} = \frac{1}{2}$ .

The dynamic scheme as well as its linearized variant have been tested a priori on a 1D sawtooth profile (figure 1, left) and a 3D turbulent field (figure 1, right), by comparing error-spectra following the work of *Chow et al.* [2]. So far, promising results are obtained. We will further systematically investigate the numerical performance of the schemes, and the impact of the improved numerics on the subgrid modelling in an a posteriori study on Large Eddy Simulations of a 1D burgers equation and a Taylor-Green Vortex Flow. We will report on this study at the conference.

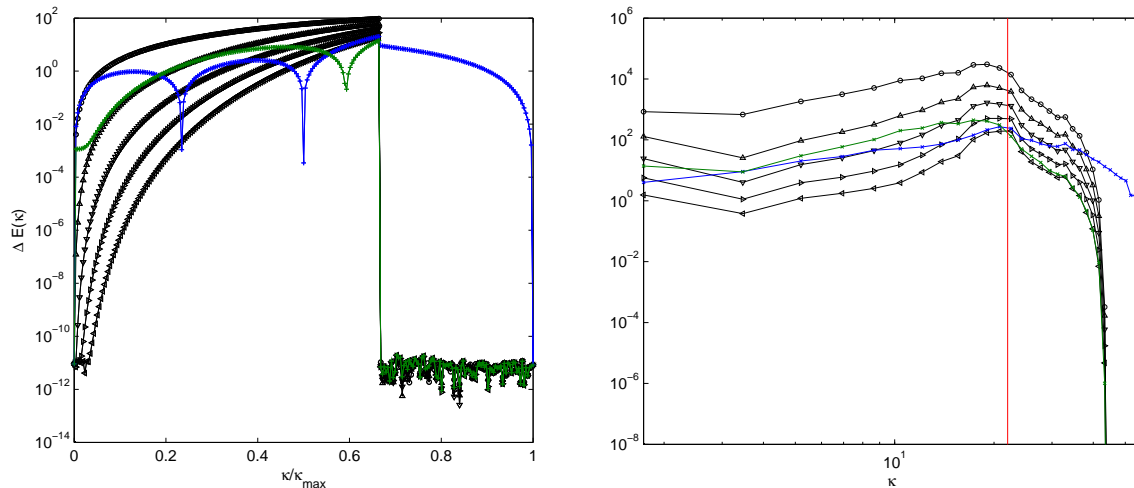


Figure 1: **Error spectrum of derivative (left) and nonlinear force (right).** ( $\circ$ ), 2<sup>nd</sup> order central; ( $\triangle$ ), 4<sup>th</sup> order central; ( $\nabla$ ), 6<sup>th</sup> order central; ( $\triangleright$ ), 8<sup>th</sup> order central; ( $\triangleleft$ ), 10<sup>th</sup> order central; ( $\times$ ), **Dynamic Scheme**; ( $\times$ ), **Linearized dynamic scheme**; ( $-$ ),  $\kappa_c = \frac{2}{3}\kappa_{max}$

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