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A DISCONTINUOUS GALERKIN METHOD FOR FREE SURFACE FLOWS WITH TOPOGRAPHY AND DRY LANDS

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ABSTRACT

A number of engineering and environmental applications are characterized by free surface flows in which the typical horizontal length scale is large compared with the vertical scale. In this case, the flow can be approximated by the shallow water equations (SWE).

Numerical simulation of free surface flows usually involves the solution of some difficult tasks, such as the capture of sharp discontinuities due to hydraulic jumps or bores in shallow flows, the flood-wave propagation over an initially dry land and its alternate recession, with consequent drying of the bottom, or the development of fast-moving flows over an irregular topography.

In last few decades, computational models based on the solution of the SWE have been developed to study a variety of free-surface flows with finite difference, finite volume and finite element methods (see, e.g. [2] and references therein). More recently, discontinuous Galerkin methods (DG) have appeared as an alternative to finite volume or high-order finite difference schemes, with a number of advantages: these allow efficient p- and h-adaptivity, local grid refining and parallel computations.

These methods have also proven to be very effective to solve the long-wave or shallow water approximation within a finite element framework [2,4]. However, issues like preservation of the steady state at rest over varying topography for higher-order approximations or the treatment of internal moving boundaries due to flooding and drying cycles in the context of DG applications are currently active subjects of research [1,2].

The preservation of the steady state at rest is a desirable property that numerical schemes must fulfill in order to satisfy the balance between the flux and the source term (when present). This property is referred in the literature as *well-balancing* and is important when computing stationary or almost stationary solutions [3]. On the other hand, flood-wave propagation (or recession) over a dry bed arises in a wide range of free-surface hydraulic problems, such as dam breaks, tidal flows or emerging bars in natural channels. One major difficulty when dealing with such a processes is to guarantee that the discrete water depth remains positive.

In this work, we propose the numerical solution of the SWE with the DG method and expanded it with numerical techniques in order to satisfy the well-balancing property over irregular topography and to be able to deal with flooding and drying cycles over dry bed. To solve the flow-at-rest issue, we derive a *flux modification* technique for DG method inspired from the hydrostatic reconstruction developed for kinetic schemes [2]. To control the nonnegativity of water depth, we introduce a *slope modification* technique, in which we ensure the presence of a thin layer of water in nominally dry areas, keeping fixed the original mesh. This technique is inspired on the idea of threshold value, usually adopted in the context of finite volume applications and has showed to be more robust and computational less expensive than, for example, mesh adaptation procedures.

The accuracy of the method is evaluated by comparison against a survey of analytical tests. In addition, the combined application of the slope limiter that control spurious oscillations around sharp fronts and the *slope modification* technique is examined. Finally, we present some practical examples to illustrate the applicability of the method. The verification and validation tests have proven that our numerical scheme is an excellent tool for modelling free-surface flows with complicated geometries and steep gradients or discontinuities in the topography. The method is also able to deal with flood-wave fronts over dry lands and its alternate recession, and to handle steady state at rest with accurate results.

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