

Finite volume simulations for the generation and propagation of long waves

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ABSTRACT

Shallow water long waves such as, tides, tsunamis and storm surges can develop large amplitudes in coastal waters with the (hazardous) potential for causing casualties and large property damage. A considerable research effort has been devoted in recent years to the development of various analytical and numerical models based on the depth-integrated, nonlinear shallow water equations (NLSWE) in order to describe the physical process. Due to the complex nature of the water motion, especially during the process of wave breaking, any simulation models of the runup of breaking long waves must be treated numerically. Numerical models can provide approximate solutions in more general settings suitable for practical applications, as predictive tools [2].

The primary goals of the present work are, (a) to apply and extend, in high-resolution and in two dimensional complex domains, a specific treatment for calculating wet/dry fronts in a well known finite volume Roe-type scheme, (b) to assess the ability of the numerical scheme to track the moving shoreline line and to (conservatively) capture discontinuities associated with bores or breaking waves, which are essential for runup calculations and, (c) to compare the computed results with analytical solutions and, more importantly, to experimental benchmark data. We note here that, using the NLSW equations solved by a finite volume scheme for the calculation of the breaking and runup of long waves was not a very common practice in the past, but in the last few years literature in this area has grown fast. The method is applied to obtain numerical simulations of (a) long wave run-up on coasts of arbitrary profile with wet/dry transitions and, (b) long wave generation and run-up due to landslides or more general due to topography changes. We present numerical results for benchmark problems with emphasis given to problems from the Third International Workshop on Long-Wave Runup Models [1].

Under certain assumptions, free-surface flow over a variable bottom topography under the influence of gravity can be modeled by the NLSWE. Based on the conservation of mass and momentum principles the equations are given as

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot \mathcal{F}(\mathbf{q}) = \mathbf{R} \quad \text{on} \quad \Omega \times [0, t] \subset \mathbb{R}^2 \times \mathbb{R}^+,$$

where $\Omega = [0, a] \times [0, b]$ and $\Omega \times [0, t]$ is the space-time domain over which solutions are sought, with

$$\mathbf{q} = \begin{bmatrix} h \\ hu \\ hv \end{bmatrix}, \quad \mathcal{F}(\mathbf{q}) = [\mathbf{F} \quad \mathbf{G}] = \begin{bmatrix} hu & hv \\ hu^2 + \frac{1}{2}gh^2 & huv \\ huv & hv^2 + \frac{1}{2}gh^2 \end{bmatrix},$$

with $\mathbf{u} = [u, v]^T$ being the vector velocity field, $h(x, y, t)$ the flow depth and g the gravitational acceleration. The source term \mathbf{R} models the effects of the shape of the bottom topography, the friction forces as well as viscous and pressure drag forces of a landslide.

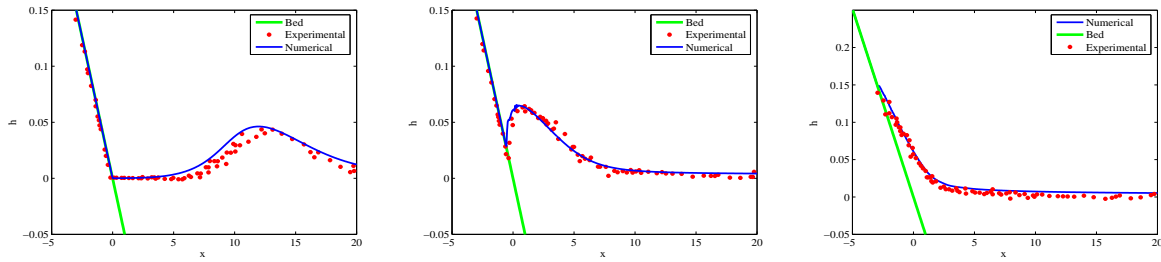


Figure 1: Time series of the water depth of a solitary wave runup on a plane beach [2]

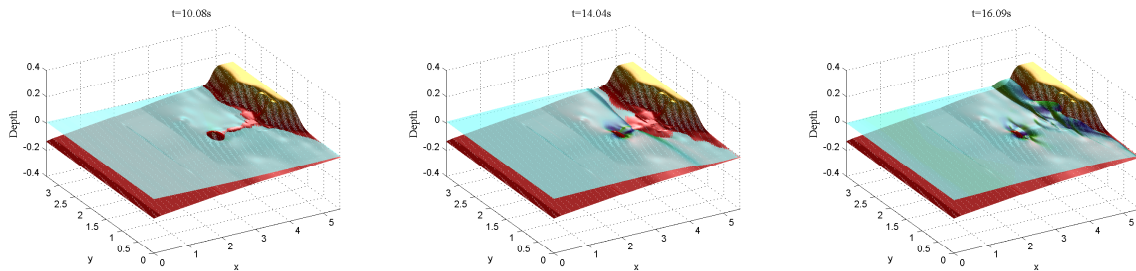


Figure 2: Run up due to the Okushiri tsunami [1]: three dimensional view time series

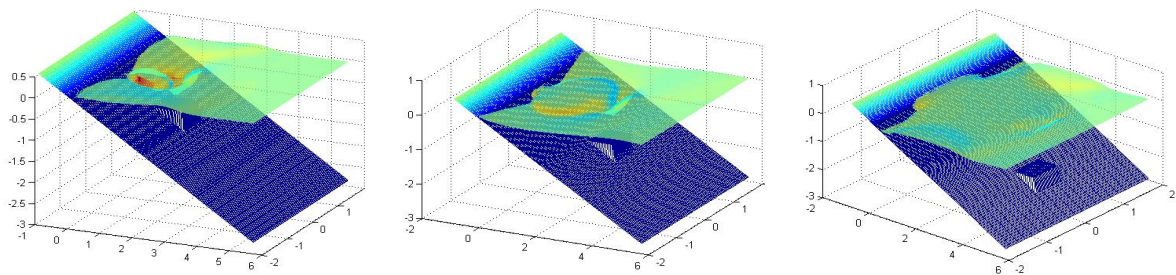


Figure 3: Time series of a 3D landslide problem [1]

The finite volume shock capturing numerical scheme that implements the Roe approximate Riemann solver and a well known definition of a well balanced conservation property (the so-called C-property), as well as a previously introduced improved numerical technique in order to calculate wet/dry fronts over adverse steep slopes, is validated and here to one and two dimensions and for a second order in space extension. Solving the NLSW equations the resulting scheme provides a reasonably good prediction of general patterns and important characteristics of the runup process and represents a breaking wave as a propagating bore or a stationary hydraulic jump showing an excellent capability in conserving the mass volume. The numerical results agree well with analytical solutions and experimental benchmark data for various problems (Figs 1-3).

REFERENCES

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