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NON-LINEAR DYNAMIC ANALYSIS OF A CABLE BASED ON HAMILTON'S PRINCIPLE AND HELLINGER-REISSNER FUNCTIONAL

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ABSTRACT

In this paper we focus on a simply supported cable, which is modelled by a linear isoparametric finite elements. Governing equations of dynamic behaviour are brought out with Hamilton's principle based on Hellinger-Reissner functional. Described algorithm was verified with an example in which a simply supported cable is taken into consideration. The load of a cable is considered as a concentrated mass in motion along a cable and dead-weight.

The form of Hellinger-Reissner functional for a general 3D problem in theory of elasticity is shown below [1]:

$$\Pi_R(\sigma^{ij}, u_k) = \int_V \left[-\frac{1}{2} D_{ijkl}\sigma^{ij}\sigma^{kl} + \frac{1}{2}(u_{i,j} + u_{j,i})\sigma^{ij} - f^i u_i \right] dV - \int_{S_\sigma} u_i \overline{p}^i \, dS_\sigma \tag{1}$$

where:
σ^{ij} – stress tensor, u_i – displacement vector, D_{ijkl} – compliance tensor,
f – volume loads, V – volume, $\overline{p}^i$ – edge loads,
S_σ – surface with known stress

Based on it we build Hamilton functional:

$$H = \delta \int_{t_1}^{t_2} (E_k - \Pi_R)\, dt = \delta \int_{t_1}^{t_2} \left(\frac{1}{2} \int_V \rho \dot{\mathbf{u}}\dot{\mathbf{u}}\, dV - \Pi_R \right) dt \tag{2}$$

where:
E_k – kinetic energy, ρ – density, $\dot{}$ – derivative over time

Deformation of a cable is described only by one component of strain tensor (elongation). We can calculate it from relationship:

$$ds^2 - ds_0^2 = 2\varepsilon ds_0^2 \tag{3}$$

and after simple transformation we obtain:

$$\varepsilon = \frac{1}{x_\xi^2 + y_\xi^2}\left[x_\xi u_\xi + y_\xi v_\xi + \frac{1}{2}\left(u_\xi^2 + v_\xi^2\right)\right] \tag{4}$$

where:
$x,\ y$ – co-ordinates on plane,
$u,\ v$ – displacements along *x* and *y* axis respectively,
$_{-\xi}$ – derivative over parameter ξ, which is local co-ordinate along the length of cable

Analysed cable is modelled by linear isoparametric finite elements described by linear shape functions for displacements. It leads to constant value of tensile force in the element. So on, the Hellinger-Reissner functional (1) takes form for an element:

$$\mathit{\Pi}_R(N, \mathbf{q}) = -\frac{1}{2}N^2\frac{l}{EA} + lN\left[\mathbf{B_0} + \mathbf{B_1}(\mathbf{q})\right]\mathbf{q} = \mathbf{F}^T\mathbf{q} \tag{5}$$

where:
N – normal force in element, l – length of element, EA – longitudinal stiffness,
$\mathbf{q}$ – vector of boundary displacements: $\mathbf{q} = [u_1;\ v_1;\ u_2;\ v_2]^T$, $\mathbf{F}$ – load,
$\mathbf{B_0},\ \mathbf{B_1}(\mathbf{q})$ – linear and non-linear (respectively) part of geometric relationship matrix

The kinetic energy is described by a formula:

$$E_k = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}\dot{\mathbf{q}} \tag{6}$$

After combining (5) and (6) with Hamilton principle (2) we obtain system of governing equations:

$$\begin{cases} -\frac{l}{EA}N + l\left[\mathbf{B_0} + \mathbf{B_1}(\mathbf{q})\right]\mathbf{q} = 0 \\ \mathbf{M}\ddot{\mathbf{q}} + l\mathbf{B_0}^T N + \frac{1}{l}[\mathbf{G}(N)]\mathbf{q} = \mathbf{F} \end{cases} \tag{7}$$

where:

$$\mathbf{G}(N) = \mathbf{G}N = \begin{bmatrix} 1 & & -1 & \\ & 1 & & -1 \\ -1 & & 1 & \\ & -1 & & 1 \end{bmatrix} N \tag{8}$$

Doing eliminations of N in top part of (7) we eventually obtain matrix equation:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}(\mathbf{q})\mathbf{q} = \mathbf{F} \tag{9}$$

where:

$$\begin{aligned} \mathbf{K}(\mathbf{q}) &= \mathbf{K}(\mathbf{q})^T = \\ &= \left[l^2\mathbf{B_0}^T\frac{EA}{l}\mathbf{B_0} + l^2\mathbf{B_0}^T\frac{EA}{l}\mathbf{B_1}(\mathbf{q}) + l^2\mathbf{B_1}^T(\mathbf{q})\frac{EA}{l}\mathbf{B_0} + l^2\mathbf{B_1}^T(\mathbf{q})\frac{EA}{l}\mathbf{B_1}(\mathbf{q})\right] \end{aligned} \tag{10}$$

and, in (9), $\mathbf{K}$ and $\mathbf{q}$ are time-dependent functions.

Doing the standard procedure of FEM aggregation we obtain system of equations for the cable. It was used Runge-Kutta algorithm for solution of equations. The examples and results would be shown during the presentation.

REFERENCES

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