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GENERAL GALERKIN ALE METHODS FOR TURBULENT FLUID-STRUCTURE INTERACTION USING A UNIFIED CONTINUUM FORMULATION

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ABSTRACT

The typical approach to fluid-structure interaction (FSI) problems is based on coupling separate discretizations of the fluid and the structure, by passing information over the fluid-structure interface as the computation progresses in time. With an explicit method the coupling is referred to as weak, and with an implicid method the coupling is strong. A weak coupling typically suffers from instabilities for many problems, whereas the strong coupling is more stable but also more expensive. In a monolithic approach solution methods are formed based on the full system with both the fluid and structure equations, which has been shown to be very robust [1].

In [2] a Unified continuum formulation (UCF) of fluid-structure interaction is proposed where where the basic conservation laws for mass, momentum and energy are fomulated for the full unified continuum of the combined fluid-structure domain, with only the constitutive laws being different. To fit with the fluid equations, the structure equations are based on an updated stress formulation [9]. An ALE stabilized finite element method is used, which we refer to as a General Galerkin (G2) method, with the structure described using Lagrangian coordinates. For incompressible fluid flow the G2 methodology has been shown to generalize to turbulent flow [3], with adaptive computation of mean value output of interest, and without using any turbulence/subgrid modeling as in RANS/LES type methods [4].

The stabilized finite element methodology for FSI is well known, see e.g. [5], and adaptivity and a posteriori error control for FSI is an active research area, see e.g. [6,7], and in this paper we present a generalization of the UCF/G2 methodology to adaptive algorithms and turbulent fluid flow.

We also present a mesh smoothing algorithm defined using an elasticity model with large displacements (a variation of the solid model in the UCF). We solve for the deformation gradient F as an unknown, which allows us to control the shape and size of each cell. This mesh smoothing allows us to solve problems with non-trivial geometries using the UCF.

The UCF is defined as:

$$\begin{aligned} \rho\dot{u} - \nabla\cdot\sigma_D - \nabla p &= f \\ 0 &= \nabla\cdot u \\ \sigma_D &= \sigma_{D,s} + \sigma_{D,f} \\ \sigma_{D,f} &= 2\mu_f\epsilon \\ \dot{\sigma}_{D,s} &= 2\mu_s\epsilon \\ \epsilon &= \frac{1}{2}(\nabla u + \nabla u^\top) \end{aligned} \tag{1}$$

where the subscript f denotes fluid and s solid, giving differing constitutive equations for the two phases, but the same momentum equation. The stress σ has been decomposed into a deviatoric part σ_D and a pressure pI.

For the FEM discretization in space, we use piecewise linear continuous functions for the velocity u and pressure p and piecewise constants for the stress σ. For the FEM discretization in time we use piecewise linear continuous functions (cG(1)). The space discretization is automatically computed using FEniCS [8] by giving the variational formulation of 1 as input.

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