8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

THE EFFECTS OF STRAIN SMOOTHING ON FINITE ELEMENT SOLUTIONS OF STRAIN SOFTENING SOLIDS

Tony Jefferson[1] **and Paul Lyons**[2]

[1] Cardiff University School of Engineering
The Parade, Cardiff, UK, CF24 3AA
JeffersonAD@cf.ac.uk, www. cardiff.ac.uk

[2] LUSAS
Forge House, 66 High Street, Kingston upon Thames, Surrey, UK, KT1 1HN
paul.lyons@lusas.com, www.lusas.com

Key Words: *Damage, Fracture, Strain softening, Regularisaion, Strain smoothing.*

ABSTRACT

Considerable work was undertaken in the 1970s and 80s on local and global stress smoothing schemes for finite element codes[1-2]. Later, various smoothing techniques were developed using mixed approaches based on either the Hu-Washizu or Hellinger-Reissener functionals [3-4]. These techniques can be used to smooth strains alone and the aim of the present work is to explore if such techniques can be used to develop a practical scheme, suitable for large scale commercial codes, to stabilise finite element solutions of strain softening materials.

Problems associated with the finite element analysis of strain softening materials have long been known and one of the early solutions to the problem of mesh size dependency was the crack band model Bazant and Oh[5]. This technique does not however necessarily improve the stability of incremental iterative solutions, which can breakdown due to the presence of alternative equilibrium paths. Two general techniques that do address both the problem of solution stability and mesh regularisation are gradient and non-local non-local methods[6-7]. However, both of these techniques would require considerable effort to implement in a general purpose code for a wide range of 2D and 3D elements and materials models.

The Hu-Washizu functional provides a very general form of the governing equation for continuum solids but Zienkiewicz and Taylor[3] demonstrated that the same discretised equations can be achieved working directly from the virtual work equation and deriving smoothed stress and/or strain fields from weak forms of the appropriate equations. In the present work only a smoothed strain field is introduced.

The governing equations in discretised form are as follows;

$$\sum_{\text{elems}} \int_{\Omega_e} \mathbf{B}^{T} \boldsymbol{\sigma}\, d\Omega_e - \sum_{\text{elems}} \int_{\Gamma_e} \mathbf{N}^{T} \mathbf{t}\, d\Gamma_e = 0 \qquad (1)$$

$$\sum_{\text{elems}} \int_{\Omega_e} \mathbf{N}^{T} (\mathbf{N}\bar{\boldsymbol{\varepsilon}} - \mathbf{B}\mathbf{u})\, d\Omega_e = 0 \qquad (2)$$

$$\boldsymbol{\sigma} = \mathbf{D}(\bar{\boldsymbol{\varepsilon}})\, \mathbf{B}\mathbf{u} \qquad (3)$$

N are the shape functions; **t,** boundary tractions; **u,** nodal displacements; $\boldsymbol{\sigma}$, stresses, **B,** strain-displacement functions; $\bar{\boldsymbol{\varepsilon}}$, smoothed strains; Ω_e, element volume; Γ_e, element surface area and **D** the constitutive matrix.

When a softening band develops in a finite element mesh, a standard application of the above smoothing equations leads to a cyclic variation of strains. It therefore proves necessary to introduce weightings into the expressions to produce an acceptable smoothing matrix. The authors[8] previously considered an application of these equations in which a global smoothing matrix was derived from weighted element level smoothing matrices, based on equation (2). The present paper examines a different form for the smoothing equations in which a set of weights, contained in the matrix α, are introduced to the smoothed strain component of equation (2) only. A set of conditions for deriving α is proposed that ensures that the smoothed strain field satisfies basic conditions such as giving zero normal strains on free boundaries and being uniform when the local strains are uniform. Furthermore, the issue of deriving the coefficients of the α matrix such that the smooth strain field is regularised is also discussed.

$$\sum_{\text{elems}} \boldsymbol{\alpha} \int_{\Omega_e} \mathbf{N}^T \mathbf{N} \bar{\boldsymbol{\varepsilon}}\, d\Omega_e = \sum_{\text{elems}} \int_{\Omega_e} \mathbf{N}^T \mathbf{B} \mathbf{u}\, d\Omega_e \qquad (4)$$

1D and 2D finite element examples are given in which a basic isotropic damage model was employed. It is shown that smoothing strain fields in the manner proposed does considerably improve the stability of strain softening finite element solutions.

REFERENCES

[1] E. Hinton and J.S. Campbell "Local and global smoothing of discontinuous finite element functions using a least squares method" . *Int. J. Num. Meth. Engng* 84, 461-480 (1974)

[2] G. Loubignac, G. Cantin and G. Touzot. "Continuous stress fields in finite element analysis". *AIAA Journal*, 15(11) 1645-1647 (1977)

[3] O.C. Zienkiewicz and R.L. Taylor, *The finite element method*, McGraw Hill, Vol. I., 1989, Vol. II, (1991).

[4] J Cervenka, S C. Keating and C.A. Felippa. "Comparison of strain recovery techniques for the mixed finite element method" *Comm. Numer Meth. In Engng*,. 9, 925-932 (1993)

[5] Z.P. Bazant and B.H. Oh. "Crack band theory for fracture of concrete". *Materials and Structures.* 16 (1983).

[6] R.H.J. Peerlings, R. deBorst, W.A.M. Brekelmans, J.H.P. deVree (1996). "Gradient enhanced damage for quasi-brittle materials" *Int. J. Num. Meth. Engng.* 39(19), 3391-3403

[7] Z.P. Bazant, G. Pijaudier-Cabot (1989). "Measurement of characteristic length of nonlocal continuum". *Journal of Engineering Mechanics ASCE.* 115(4), 755-767

[8] A.D. Jefferson and P. Lyons (2007) "Stability and regularisation of finite element solutions of damage problems with strain smoothing", *Proceedings of COMPLAS 2007.*