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FAST MULTIPOLE COMPRESSION FOR NON-HOMOGENOUS PARTS OF POISSON TYPE PROBLEMS

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ABSTRACT

Our research group works on the application of the Boundary Element Method to fluid flow problems. Recently Ravnik et al. [4] developed a 3D subdomain - single domain BEM numerical scheme for solving incompressible velocity-vorticity formulation of Navier-Stokes equations. A crucial part of this algorithm is the calculation of boundary vorticity values. They are obtained by solving a Poisson type partial differential equation using single domain BEM. The domain contribution, which arises from the non-homogenous part of the Poisson equation, requires the discretization of the domain and the calculation of a large number of domain integrals, thus limiting the maximal mesh size.

An integral form of Poisson type equation for a scalar field function $u(\vec{r}) \in \Omega$ is

$$c(\vec{\xi})u(\vec{\xi}) = \int_\Gamma \left[u^\star(\vec{\xi},\vec{r})\vec{\nabla}u(\vec{r}) - u(\vec{r})\vec{\nabla}u^\star(\vec{\xi},\vec{r}) \right] \cdot d\vec{\Gamma} - \int_\Omega u^\star(\vec{\xi},\vec{r})b(\vec{r})d\Omega \qquad \vec{\xi} \in \Gamma, \tag{1}$$

where $\vec{\xi}$ is the collocation point on the boundary and $u^\star = 1/4\pi|\vec{r} - \vec{\xi}|$ is the fundamental solution of the Laplace equation in 3D. When the collocation point $\vec{\xi}$ is set to all boundary nodes, a system of linear equations is obtained. Complexity of such a numerical scheme can be estimated. The boundary integrals are stored in full unsymmetrical matrices. Their number scales as $\mathcal{O}(n_b^2)$, where n_b is the number of boundary nodes. The domain integral matrix scales as $\mathcal{O}(n_d \cdot n_b)$, where n_d is the number of nodes in the domain. Considering a cube, one can estimate $n_d = n^3$, $n_b \approx n^2$, thus the complexity of the domain integral matrix is $\mathcal{O}(n^5)$, where n is the number of nodes on one edge of our model cube, while the boundary matrix scales as $\mathcal{O}(n^4)$. Since, clearly, the domain contribution takes up most of the CPU time and storage space, this paper presents an application fast multipole method (Greengard and Rokhlin [1], Hackbusch and Nowak [2]) to compress the domain matrix in such a way that the number of non-zero elements scales nearly linearly $\mathcal{O}(n_d \log n_d)$.

The method is based on the fact that it is possible to expand the fundamental solution into a series and by doing so, separate the variables, i.e. the collocation point $\vec{\xi}$ and the domain point $\vec{r}$. In this work we use spherical harmonics

$$\frac{1}{4\pi|\vec{r} - \vec{\xi}|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{(-1)^m}{2l+1} \frac{1}{\xi^{l+1}} Y_l^{-m}(\theta_\xi, \varphi_\xi) r^l Y_l^m(\theta_r, \varphi_r) \tag{2}$$

with $\vec{r} = (r, \varphi_r, \theta_r)$ and $\vec{\xi} = (\xi, \varphi_\xi, \theta_\xi)$. The expansion enables calculation of matrix blocks ($n_{row} \times n_{col}$) by multiplying two small matrices ($n_{row} \times n_{exp}$) and ($n_{row} \times n_{exp}$), where n_{exp} is the number of expansion terms, yielding compression of a block.

The procedure works for matrix blocks, were expansion is successful for the entire range of collocation points and the entire domain, which correspond to a particular block of a matrix. Since the collocation points are located on the boundary and the integral is calculated over a domain, we must divide both boundary and domain into blocks. Recursively both boundary and domain cluster trees are built. Since each matrix block corresponds to a part of the boundary and a part of the domain, the boundary-domain cluster tree is formed by combining the clusters from the boundary and domain trees.

In order to decide which combination of boundary and domain clusters (a node in the boundary-domain tree) may be used to approximate the integral kernel by expansion, we require an admissibility criterion. For each node in the cluster tree we calculate the coordinate origin, which enables best compression (best r/ξ ratio). Admissibility criterion is compared against the largest expansion error of all boundary-domain node pairs in the cluster. The parts of the matrix, where the admissibility condition can not be satisfied, are stored and calculated as full matrix blocks. An example of a division of a matrix to admissible and inadmissible blocks is shown in Figure 1.

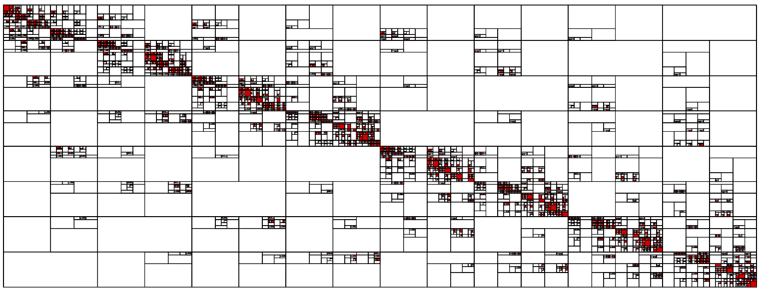

Figure 1: Domain integral matrix structure of a cubic mesh (33^3 nodes). Red areas show inadmissible parts of the matrix, white areas are admissible.

The developed numerical scheme enables computation of boundary vorticity values with nearly linear complexity of the domain contribution. It is a part of a 3D BEM velocity-vorticity flow solver. It decreases the CPU time and storage requirements for the domain integral matrix and enables simulations of fluid flow on denser meshes. This type of compression does not require the calculation of the full matrix at any time, which is an advantage over similar wavelet based schemes (Ravnik et al. [3]).

References

[1] L. Greengard and V. Rokhlin. A fast algorithm for particle simulations. *J. Comput. Phys.*, 73:325–348, 1987.

[2] W. Hackbusch and Z. P. Nowak. On the fast multiplication in the boundary element method by panel clustering. *Numerische Mathematik*, 54:463–491, 1989.

[3] J. Ravnik, L. Škerget, and M. Hriberšek. The wavelet transform for BEM computational fluid dynamics. *Eng. Anal. Bound. Elem.*, 28:1303–1314, 2004.

[4] J. Ravnik, L. Škerget, and Z. Žunič. Combined single domain and subdomain BEM for 3D laminar viscous flow. *Eng. Anal. Bound. Elem.*, submitted:xx–xx, 2008.