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POST-BUCKLING BEHAVIOUR OF A COMPRESSED SLENDER BEAM CONSTRAINED TO A CYLINDRICAL TUBE

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ABSTRACT

The paper deals with the study of the behaviour of a slender beam introduced in a cylindrical tube and subjected to an axial compressive force and to a torque. The beam is very long compared to its transversal dimensions and therefore, it will buckle to very small axial force according to the Euler formula. The post-buckling behaviour is examined, without and with friction. The study has important applications in the buckling of drill strings inside a cylindrical hole in the case of deep drilling, rotary drilling of wells in bedrocks at depths of several kilometres, buckling of a homogeneous cable in a horizontal circular rigid duct subjected an axial compressive load (for instance posing of optical fibre), in petroleum industry, for coiled tubing, especially in the case of drilling in horizontal or inclined wellbores, etc, [1-4].

The slender beam has a constant cross-section that can have any form, although circular cross-section is the most used in practice. The problem has a geometrical non-linearity to which the non-linearity caused by the friction has to be added. A special isoparametric 3D beam finite element is elaborated. The paper presents only the static case, but extending the presented approach to dynamic analysis is quite natural and it will be presented in another work. The method is very accurate and very rapidly convergent due to the fact that the exact equations, that is written for the deformed configuration, are solved. Small strains and small displacements are assumed as the diameter of the cylindrical tube is much smaller than the length of the beam. Material is considered linear. Large twist angles could occur as an exception, but the specific twist angle is small. The iterative Newton-Raphson method was used to solve the nonlinear differential equations. To characterize the deformed shape of the beam the Euler-Rodrigues quaternion is used: $\boldsymbol{l} = (l_0 \quad l_1 \quad l_2 \quad l_3)$ with $\boldsymbol{l}^T \boldsymbol{l} = 1$, [5]. In the case of small displacements l is: $\boldsymbol{l} = (1 \quad \varphi_x/2 \quad \varphi_y/2 \quad \varphi_z/2)$ where $\varphi_x, \varphi_y, \varphi_z$ are the components of the total rotation angle $\boldsymbol{\varphi}$, considered as vector, in the global reference

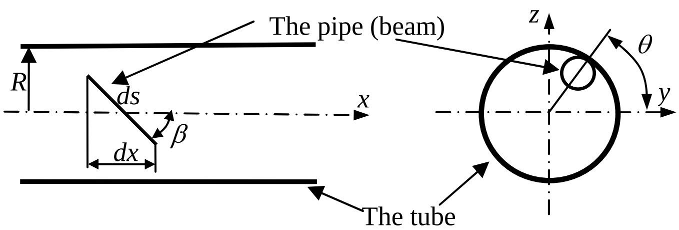

Figure 1. Definition of angles

frame *xyz* (see figure 1). The angles β and θ of the helical post-buckling configuration, figure 1, and angles φ_y, φ_z, characterising an infinitesimal beam element d*s* are linked by the equations:

$$\begin{cases} \varphi_z = -\beta \sin\theta \\ \varphi_y = -\beta \cos\theta \end{cases} \text{ and } \beta = R\frac{d\theta}{ds} \qquad \text{or finally} \qquad \begin{cases} \varphi_z = -R\dfrac{d\theta}{ds}\sin\theta \\ \varphi_y = -R\dfrac{d\theta}{ds}\cos\theta \end{cases}$$

Nodal unknowns are: ε_0, φ_x and θ, that is three DOFS/node, where ε_0 is the axial strain. Total potential energy theorem is applied; curvature vector:

$$\boldsymbol{\kappa} = \left(\frac{d\varphi_x}{ds} + \frac{1}{2}\left(\varphi_z \frac{d\varphi_y}{ds} - \varphi_y \frac{d\varphi_z}{ds}\right) \quad \frac{d\varphi_y}{ds} + \frac{1}{2}\left(\varphi_x \frac{d\varphi_z}{ds} - \varphi_z \frac{d\varphi_x}{ds}\right) \quad \frac{d\varphi_z}{ds} + \frac{1}{2}\left(\varphi_y \frac{d\varphi_x}{ds} - \varphi_x \frac{d\varphi_y}{ds}\right) \right)^T$$

and linear displacements are expressed in the nodal unknowns. The displacements of two consecutive nodes are given by equation (the integral is computed numerically using the shape function):

$$\begin{Bmatrix} du_{i+1} \\ dv_{i+1} \\ dw_{i+1} \end{Bmatrix} = \begin{Bmatrix} du_i \\ dv_i \\ dw_i \end{Bmatrix} + \int_i^{i+1} \left(\begin{bmatrix} 0 & -\varphi_y & -\varphi_z \\ \frac{1}{2}\varphi_y & \frac{1}{2}\varphi_x & 1 \\ \frac{1}{2}\varphi_z & -1 & \frac{1}{2}\varphi_x \end{bmatrix} \begin{Bmatrix} d\varphi_x \\ d\varphi_y \\ d\varphi_z \end{Bmatrix} \right) (1+\varepsilon_0) ds$$

Figure 2 shows an example of a compressed beam having a circular annular cross section, constrained in a cylindrical tube. All the data are given in the figure.

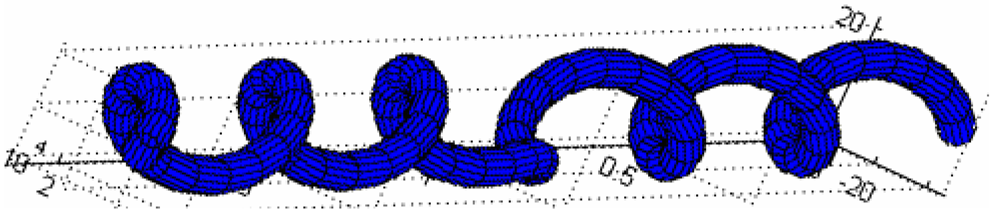

Figure 2. Post-buckling configuration of a compressed beam: *F*=2250 N, *L*=21340 mm, *R*=24.13 mm, annular cross section of the beam *d*=6.35 mm; *D*=13.72 mm

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