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A NEW DISCONTINUOUS GALERKIN METHOD FOR THE NAVIER-STOKES EQUATIONS

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ABSTRACT

For the Navier-Stokes equations discretized by a Galerkin (DG) method, the viscous fluxes are defined through the gradients of primitive variables. Since the DG solution is discontinuous at cell faces, an adequate treatment is required for these gradients. At the 16^{th} ICNMFD, Arcachon,1998 [1], we presented a comparison between two formulations to discretize the viscous terms with DG : the first one uses the Green formula on a shifted cell and the second one is the mixed formulation proposed by Bassi and Rebay [2] in which an auxiliary equation for the viscous gradients is solved with the same DG approximation as for the inviscid terms. The previous shifted cell technique is inadequate for high order DG-P_k method with $k > 1$, as being only of second order of accuracy. What we propose here is to extend this technique by integrating the same auxiliary equation as for the mixed formulation but by integrating it on a shifted cell built by agglomerating the two adjacent cells with a DG-P_k method. The previous technique can be interpreted in terms of a DG-P_0 discretization. Our new method though different is related to the recovery method proposed by Van Leer [3] in which a higher order smooth solution is recovered from the DG solution on the same shifted cell. The advantage of this formulation over the mixed formulation or the recovery technique is that, now, the viscous fluxes of the Navier-Stokes equations at the cell faces are perfectly defined.

A Finite-Volume Space Time Discontinuous Galerkin (DG) solver has been developed for the 3D Euler and Navier-Stokes equations on structured AMR hexahedral grids ([4], [5]). Results on a variety of academic test cases such as laminar flat plate, laminar flow over the NACA0012 profile or over the ONERA M6 wing (Fig.1,Fig. 2, inviscid case) and turbulent flow over a cavity wing (Fig. 3)will allow to evaluate this new formulation.

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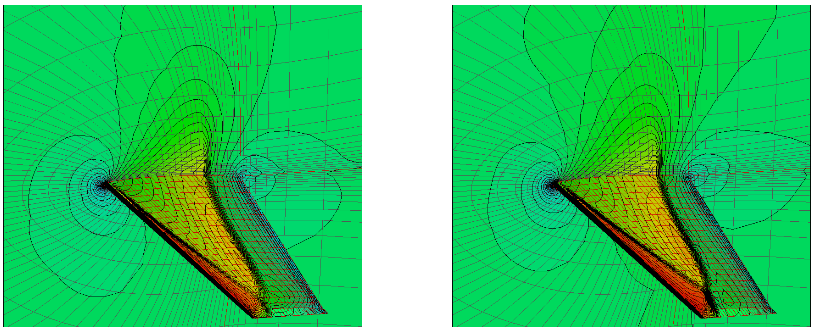

Figure 1: Transonic Flow over the ONERA M6 wing : MUSCL (left) and DG-P_1 computation (right)

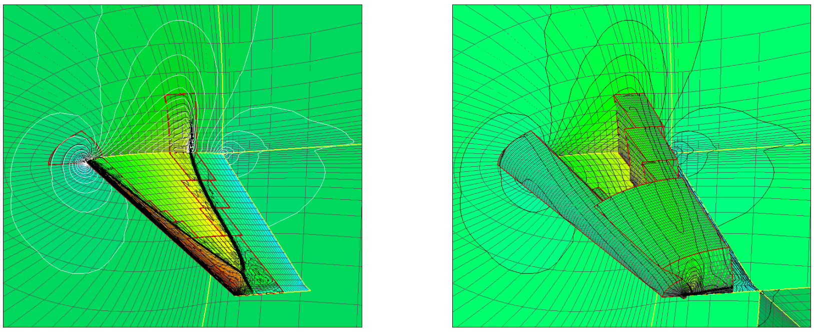

Figure 2: AMR computation

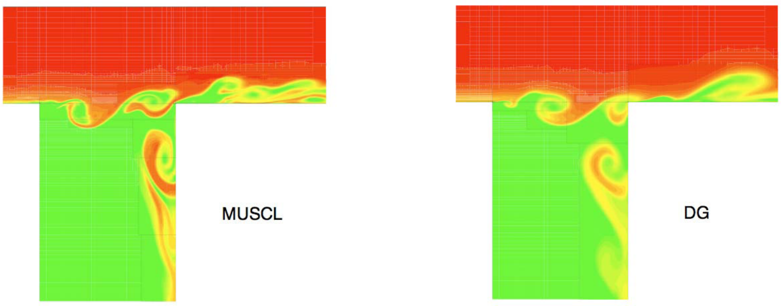

Figure 3: Turbulent flow over a cavity : entropy isovalues, MUSCL (left) and DG-P_1 computation (right)