8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

SUBSTITUTE THERMAL CAPACITY OF ALLOY. AN INVERSE PROBLEM SOLUTION

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Key Words: *Inverse Problems, Mathematical Model of Solidification, Numerical Methods.*

ABSTRACT

In the paper the problem of substitute thermal capacity $C(T)$ identification is discussed. This very convenient on a stage of numerical modelling parameter allows to describe the thermal processes proceeding in the non-homogeneous domain of solidifying metal using only one Fourier-type energy equation (a one domain method [1]). Substitute thermal capacity can be described in different ways, and here we consider its approximation by a staircase function. The values of successive 'stairs' are assumed to be unknown, at the same time they correspond to molten metal, mushy zone and solid body. The mushy zone subdomain is situated between border temperatures T_S and T_L resulting from equilibrium diagram. On the basis of additional information concerning the cooling curves at the selected set of points from metal domain the unknown parameters can be found. The inverse problem is solved using the least squares criterion in which the sensitivity coefficients are applied. On the stage of numerical simulation the boundary element method is used. In the final part of the paper the example of computations are shown.

The energy equation describing the casting solidification is the following [1, 2]

$$c(T)\frac{\partial T(x,t)}{\partial t} = \nabla\left[\lambda(T)\nabla T(x,t)\right] + L\frac{\partial f_S(x,t)}{\partial t} \tag{1}$$

where $c(T)$ is a volumetric specific heat, $\lambda(T)$ is a thermal conductivity, L is a volumetric latent heat, f_S is a volumetric solid state fraction at the considered point from metal domain, T, x, t denote the temperature, geometrical co-ordinates and time. The form of equation (1) shows that only conductional heat transfer is considered and the convection in the molten metal subdomain is neglected.

On the external surface of the system the boundary conditions, while for time $t = 0$ the initial condition are given.

In the case of typical macro model of alloy solidification basing on the one domain concept, the knowledge of temperature-dependent function f_S in the mushy zone $T \in [T_S, T_L]$ sub-domain is assumed to be known, and then (after the simple mathematical manipulations) one obtains

$$\left[c(T) - L\frac{\mathrm{d}f_S}{\mathrm{d}T}\right]\frac{\partial T(x,t)}{\partial t} = \nabla\left[\lambda(T)\nabla T(x,t)\right] \tag{2}$$

where $c(T) - L\mathrm{d}f_S/\mathrm{d}T = C(T)$ is a substitute thermal capacity [1]. One can see that for $T > T_L$: $f_S = 0$, while for $T < T_S$: $f_S = 1$ and then $\mathrm{d}f_S/\mathrm{d}T = 0$. So, the equation (2) determines the thermal processes in the whole, conventionally homogeneous casting domain. If the linear course of $f_S(T)$ for $T \in [T_S, T_L]$ is assumed, then the function $C(T)$ can be in approximated by a staircase function [3].

Let us assume that the parameters appearing in the mathematical model of alloy solidification are known except the segments creating the function $C(T)$. In order to solve the parametric inverse problem discussed, the knowledge od values T_{gi}^f at the selected set of points x_i (sensors) from metal domain for times t^f is necessary.

In order to solve the inverse problem the least squares criterion is applied

$$S(C_1, C_2, C_3) = \frac{1}{MF}\sum_{i=1}^{M}\sum_{f=1}^{F}\left(T_i^f - T_{gi}^f\right)^2 \tag{3}$$

where $T_i^f = T(x_i, t^f)$ are the temperatures being the solution of direct problem for assumed set of parameters at points x_i, $i = 1, 2, ..., M$, for time t^f, while the parameters creating the successive stairs are denoted by C_e, $e = 1, 2, 3$.

Differentiating the criterion (3) with respect to unknown parameters C_e and using the necessary condition of minimum one obtains the system of equations and using a simple iterative procedure the parameters C_e can be determined [2]. The basic problem and sensitivity ones have been solved using the BEM [3].

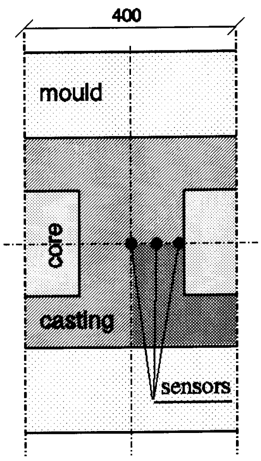

Figure 1: Domain considered

As an example the recurrent fragment of steel casting produced in a typical sand mould has been considered (Fig. 1). The position of sensors is marked in the same Figure. The start point of iteration process corresponds to C_{e0} = 10 MJ/m^{3}K and the final results are close to the assumed ones, this means $C_1 = 5.9$, $C_2 = 62.1$, $C_3 = 4.9$. The iteration process was convergent and the same results have been obtained for the others values of C_{e0} .

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