

Goal-oriented error estimation and adaptivity for fluid-structure interaction

* Kristoffer G. van der Zee¹, Harald van Brummelen¹, Ido Akkerman¹ and René de Borst²

¹ Aerospace Eng., Delft Univ. of Technology
P.O. Box 5058, NL-2600 GB Delft
{k.g.vanderzee, e.h.vanbrummelen, i.akkerman}@tudelft.nl
www.lr.tudelft.nl/em

² Mech. Eng., Eindhoven Univ. of Techn.
P.O. Box 513, NL-5600 MB Eindhoven
r.d.borst@tue.nl
www.tue.nl/wtb

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ABSTRACT

Numerical simulations of fluid-structure interaction typically require vast computational resources. Finite-element techniques employing *goal-oriented adaptive* strategies could offer a substantial improvement in the efficiency of such simulations. These strategies rely on a-posteriori error estimates for specific output quantities of interest, the goal functionals. Considerable work on goal-oriented error estimation and adaptation for generic boundary value problems has been performed by Becker and Rannacher, Oden and Prudhomme, a.o.; see [1,2,3].

The *free-boundary character* of fluid-structure-interaction problems forms a fundamental complication within the standard a-posteriori error-estimation framework, as it yields the underlying fluid-domain unknown a-priori. Consequently, the formulation of an appropriate *linearized dual* (or adjoint) problem is nontrivial [4,5]. The main difficulty is associated with the linearization of the fluid equations with respect to structure perturbations. In principle, these *domain* derivatives have two analogues in classical shape optimization: *material derivatives* and *shape derivatives*. Both derivatives yield appropriate dual problems which can be used to obtain goal-oriented error estimates for fluid-structure-interaction problems. We note that this approach does not involve a total Eulerian framework as considered in [6].

As a model problem, we consider incompressible flow past a backward-facing step with a flexible bottom, see [7]. We apply the estimator in an h-adaptive refinement strategy to control the discretization error in various quantities of interest. This is compared with respect to uniform refinement and adaptive refinement based on the energy norm of the system.

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