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A NUMERICAL STUDY ON THE STABILITY OF THE STEADY DISPLACEMENT OF A LIQUID PLUG ALONG A SMALL CONDUIT

Diego M. Campana[1,2], Sebastián Ubal[1,2], María D. Giavedoni[1] and Fernando A. Saita[1]

[1] Instituto de Desarrollo Tecnológico para la Industria Química, UNL-CONICET
Güemes 3450, 3000 Santa Fe, Argentina
madelia@santafe-conicet.gov.ar

[2] Facultad de Ingeniería, UNER
Ruta Prov. No. 11 KM 10, 3101 Oro Verde, Entre Ríos, Argentina
dcampana@santafe-conicet.gov.ar

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ABSTRACT

Liquid plugs are commonly encountered in a large number of technological applications such as oil recovery and micro-channel reactors; also, they may form in the respiratory tree either naturally from an instability of the liquid film lining the walls of the smallest conduits during the expiration process in certain pathological conditions or from the instillation of a liquid for therapeutic purposes.

A prototype of this problem is the motion of a certain volume of liquid inside a capillary tube coated by a film of the same fluid. When the propagation is steady, the thickness of this film (the precursor film) must be equal to the thickness of the film left by the bubble travelling behind the plug (the deposited or trailing film). This steady motion might be the result of drawing out the front bubble with constant velocity or, more commonly, of applying a constant pressure drop between the front and the rear gas phases.

When the liquid plug is large, the gas phases can be regarded as semi-infinite bubbles travelling alone and the flow problem can be split into two smaller ones (one for the leading and the other for the trailing bubble); these problems are linked by the film thickness which is solely determined by the leading bubble.

The steady motion of a semi-infinite bubble in a capillary tube or between two closely spaced parallel plates initially filled with a liquid, has been extensively studied analytically, numerically, and experimentally since the pioneering works by Taylor[1] and Bretherton.[2]

When the distance between the bubble tips is small, the propagation of the plug is affected by the interaction of the gas phases, and the velocity and pressure fields must be simultaneously computed in the whole domain (the central core region and the rear and trailing menisci). The works on this subject are considerably less numerous and many of them were carried out at the group leaded by Professor Grotberg, mainly motivated by the transport of liquid plugs in the pulmonary airways. These studies theoretically or numerically investigated the effects of propagation speed[3],

surfactants[4,5] and gravity[5] on the motion and splitting of a liquid plug.

An important point of one of these works[3] concerns the stability of the steady states computed. In fact, the authors conjecture that the lack of convergence of their numerical algorithm within certain range of the parameters might be due to the non existence of stable steady states; i.e., if the steady state solution is perturbed, the distance between the menisci would either continuously increase or decrease until the collapse of the plug.

The aim of this work is to analyze the stability of the steady state displacement of a liquid plug. We study two particular situations: (a) the leading gas phase is forced to move at a constant speed or, (b) a constant pressure difference between the bubbles drives the motion of the plug.

In order to conduct the study, the numerical solution of the Navier-Stokes equations is required. We employed an algorithm based on the Galerkin/finite element method combined with the parameterization of the free surface by means of spines for the spatial discretization of the governing equations and their boundary conditions. A finite difference scheme and an automatic step-size control are used to march on time.[6]

Graphs of the steady state dimensionless film thickness (H_∞), as a function of the dimensionless plug length (L_P) within a large range of the parameters, are built. We show by simple physical arguments that the stability of the system may be inferred from the shape of these curves; these results were verified by performing transient simulations of the system. For case (a), we found a critical Reynolds number beyond which the steady state solutions are unstable. In case (b), the steady state solutions are unstable except in bounded regions in the H_∞–L_P plane, located for values of $L_P \leq 1$ approximately.

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