8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Decomposition Techniques for Global Optimization of Discrete Topology Design Problems

*** E. Muñoz[1], M. Stolpe[2] and M.P. Bendsøe[3]**

[1] Dept. of Mathematics
Technical University
of Denmark (DTU)
Matematiktorvet, B. 303S
DK – 2800 Kgs. Lyngby
Denmark
E.Munoz@mat.dtu.dk

[2] Dept. of Mathematics
Technical University
of Denmark (DTU)
Matematiktorvet, B. 303S
DK – 2800 Kgs. Lyngby
Denmark
M.Stolpe@mat.dtu.dk

[3] Dept. of Mathematics
Technical University
of Denmark (DTU)
Matematiktorvet, B. 303S
DK – 2800 Kgs. Lyngby
Denmark
M.P.Bendsoe@mat.dtu.dk

Key Words: *Topology Optimization, Global Optimization, Integer Programming.*

ABSTRACT

We study a mixed-integer formulation of the minimum compliance problem for structural topology design ([2]) from both the theoretical and numerical points of view. Previously, it has been numerically indicated that the original discrete formulation of the minimum compliance problem cannot be efficiently solved when the number of design variables becomes large ([1]). Here we propose to solve this class of problems by using two different, but similar, decomposition techniques for mixed-integer optimization. The methods solve this kind of problems to global optimum and allow us to considerably increase the number of design variables. These techniques have not been used before in the area of structural topology optimization.

The discrete design problem is set in a format based on a finite element discretization of the continuum problem. Now, if we have n discrete design variables, it follows that the vector of design variables is expressed as $x \in \{0,1\}^n$. This notation describes the material distribution on the design space. The state variable $u \in \mathbb{R}^d$ represents the displacement of the structure for its d degrees of freedom, when the structure is under a single load condition $f \in \mathbb{R}^d$, and suitable support conditions. $K(x) \in \mathbb{R}^{d\times d}$ denotes the global stiffness matrix. Then the discrete formulation of the minimum compliance problem constrained to a certain capacity (weight or volume) of material V is given by

$$\begin{array}{ll} \min\limits_{x\in\mathbb{R}^n, u\in\mathbb{R}^d} & f^T u \\ \text{s.t.} & K(x)u = f \\ & \sum\limits_{j=1}^{n} x_j \leq V \\ & x_j \in \{0,1\} \qquad j = 1,...,n. \end{array} \tag{1}$$

where $K(x)u = f$ is the state equilibrium equation for the structure under the load f. This kind of problems belongs to the class of mixed integer non convex problems.

The decomposition techniques we consider are, the Generalized Benders Decomposition in its extended version ([4]), and the Outer Approximation method ([3]). These techniques are based on the fact that the non-linear mixed-integer program (1) can be replaced by the decomposed set of programs

$$z = \min_{x \in \{0,1\}^n} \{v(x) \quad \text{s.t.} \quad \sum_{i=1}^{n} x_i \leq V\} \tag{2}$$

where $v(x)$ is defined by the linear program

$$v(x) = \inf_{u \in \mathbb{R}^d} \{f^T u \quad \text{s.t.} \quad K(x)u = f\}. \tag{3}$$

The first program is called ”the master problem”, and the second problem is called ”the subproblem”. The subproblem (3) corresponds to a displacement analysis of the given structure described by x. This decomposed representation is based on the projection theory for general non-linear optimization programs. The two decomposition techniques are mainly based on finding, each one in a different way, two sequences of optimization programs, that approximate the system (2) - (3). The approximation is understood in the sense that it produces convenient bounds for the original program (1). In practice, the subproblem (3) does not include integer variables, therefore it is tractable to solve numerically, and when it is solved, it gives an upper bound for the global optimum. Hence, it does not need to be replaced, or approximated by another program. On the other hand, the program (2) is an unknown (because $v(x)$ is implicitly defined), non-linear integer problem, so it is necessary to find a convenient sequence of linear integer approximations, called relaxed master problems. As a linear integer problem, a relaxed master problem can be solved with any method for linear integer optimization, as for example, the branch and cut method. The problems produce a sequence of lower bounds for the optimum solution of (1). Therefore, this system of relaxed master problem - subproblem, forms a sequence of lower and upper bounds for the global optimum of the original problem (1). In any stage of the algorithm where the relaxed master problem is solved, a new linear constraint is introduced to the program, which guaranties that the sequence of optimal solutions for the relaxed master problems forms an increasing monotone sequence. As a consequence, the algorithm forms a monotone sequence of lower and upper bounds of the global optimum, which can be proven to converge in a finite number of iterations to the global optimum of the program (1).

The theory for the convergence of these two techniques exists since the end of the 80’s, and it is valid for a general set of optimization problems. Nevertheless, no numerical tests have been done on structural or topology optimization problems, until now. Both methods are investigated by computational means, using the standard finite element method to solve the projected subproblem, and the so-called branch and cut method for the linear integer master problem. Theoretical results of both techniques applied to the single load minimum compliance problem will be presented, as well as the numerical results. We will also present a comparison of the two techniques on benchmark examples of the structural optimization field. Finally, we will present, the future work that the results of this work suggest.

REFERENCES

[1] W. Achtziger and M. Stolpe. “Truss Topology Optimization with Discrete Design Variables - Guaranteed Global Optimality and Benchmark Examples”, *Structural and Multidisciplinary Optimization*, Vol. **34(1)**, 2007.

[2] M.P. Bendsøe and O. Sigmund. *Topology Optimization, Theory, Methods and Applications*, 2nd Edition, Springer-Verlag, 2002.

[3] R. Fletcher and S. Leyffer. “Solving mixed integer non-linear programs by outer approximation”, *Mathematical Programming*, Vol. **66**, 327–349, 1994.

[4] R. Lazimy. “Extension of the generalized Benders Decomposition”, *Communications on Applied Numerical Methods*, Vol. **6**, 195–203, 1986.