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The Linear Sampling Method and Eigenfunctions of the Far Field Operator

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ABSTRACT

The Linear Sampling method is a by now well-established method for the qualitative reconstruction of obstacles from far field data in time-harmonic inverse scattering problems. In contrast to various iterative methods, no forward problems need to be solved. The reconstruction is obtained entirely from the spectral data of a certain linear integral operator called the *far field operator.* See [1] for a recent monograph on the subject.

In its original form, although the method reliably provides good reconstructions for various inverse scattering problems, a rigorous mathematical analysis is missing. In the talk, a variant of the method will be presented, for which convergence of the reconstructions to the true obstacle can be proved when the noise level goes to zero. The approach is based on connections between the Linear Sampling method and the related *Factorization method* (see [2]). The main theorem presented in the talk gives an equivalence between reconstructions obtained by both methods for certain classes of scattering problems.

These approaches make use of eigenfunction expansions of the far field operator. Some recent research [3] has shown that Herglotz wave functions generated by these eigenfunctions can be used directly for imaging of the unknown obstacle. More precisely, Herglotz wave functions with densities from the orthognal complement of certain eigenfunctions are non-scattering waves. We investigate this phenomen further and report on some new results in this direction.

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