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# Stabilized FEM for compressible-incompressible interface flows

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## ABSTRACT

The present work is devoted to the study on unsteady flows of two immiscible viscous fluids separated by a moving interface. Especially we consider flows with both gas and liquid phases. This study has diverse environnemental and engineering applications such as bubble dynamics, channel flows, etc.

From a physical point of view, several approaches are available to modelize such flows. Classically both phases are assumed either compressible [1] or incompressible [2]. These two approaches have some drawbacks related to the inherent physical properties of each phase. For example in the first one, the volume conservation of the incompressible part is not preserved and in the second one the changes of volume of the compressible part can not be taken into account.
Here, we investigate a third approach which consider the gas as compressible and the liquid as incompressible [3].

From a numerical point of view, most methods for fluids flows are developped either for compressible flow or incompressible flow. In [3], a numerical scheme was developped to treat the coupling of compressible and incompressible flows, but each phase is solved with different methods, a TVD finite volume scheme for the compressible phase and a MAC scheme for the incompressible one, leading to a non uniform numerical scheme. Our goal is to elaborate a unified approach which is valid for both types of flow. The concept of symmetrization using entropy variables offers a good starting point towards a unified strategy. This approach allows good asymptotic behaviour between compressible and incompressible limit both for the set of primitive variables and entropic variables [4], and permits to construct efficient stabilized operators for stabilized finite element method that present the same properties [5].

In our procedure we propose using the primitive variables and we elaborate a strategy for numerical modeling of two-phase flows that essentially relies on two basic components :

- the stabilized finite element for spatial approximation of Navier-Stokes equations,

- the Level Set method for tracking precisely the interface with a discontinuous Galerkin method to solve the associated transport equation.

We focus on the specific treatment of the interface. Especially we have to design stabilization operator. In order to adress this difficulty, we propose two approaches : an “average approach” and a generalization of the “Ghost fluid method” for unstructured finite element method for compressible-incompressible.

These strategies are performed on both one and two spatial dimensions. In the two dimensions case, we consider an incompressible droplet of water moving to the right in a domain full of compressible air initially at rest. The droplet traveling causes a compression wave in the gas ahead of it and an expansion wave in the gas behind it. Figure 1 shows the pressure on a one-dimensional cross section of these waves and the velocity field through the moving interface. These results are similar to those presented in [3], that allow us to validate the stabilized finite element approximation of the coupled Navier-Stokes equations and the design of averaged terms. Nevertheless, the use of a Level-Set method to track the interface in two dimensions is still under validation.

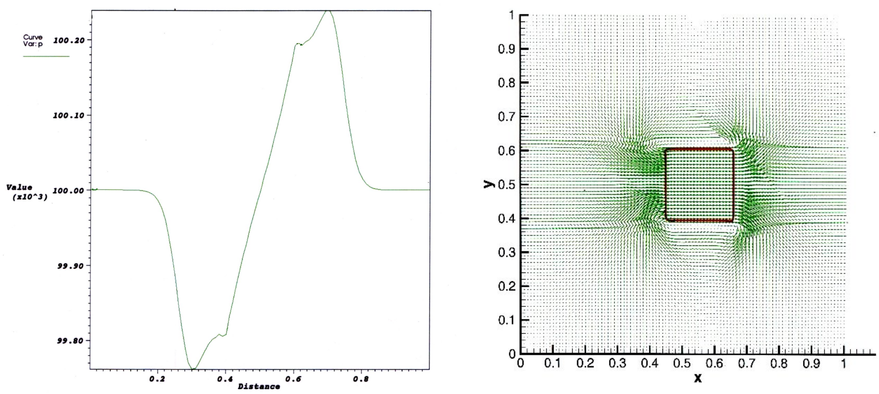


Figure 1: Pressure on a one-dimensional cross-section (left), Velocity field and interface (right)

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