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HOMOGENIZATION OF THE FIBER-REINFORCED COMPOSITES UNDER STOCHASTIC AGEING PROCESS

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ABSTRACT

The main issue of this paper is a determination of the homogenized elastic properties of the periodic fiber-reinforced composite materials [1], where Young moduli of the constituents are subjected to ageing process and are approximated by the additional stochastic processes [2]. The homogenization problem related to the rectangular Representative Volume Element Ω with the boundary Γ is given here by the set of the following stochastic partial differential equations:

$$\begin{cases} \sigma_{ij,j}^{(pq)}(\mathbf{x};\omega;t)=0 \\ \varepsilon_{ij}^{(pq)}(\mathbf{x};\omega;t)=\frac{1}{2}\left(\chi_{i,j}^{(pq)}(\mathbf{x};\omega;t)+\chi_{j,i}^{(pq)}(\mathbf{x};\omega;t)\right) \\ \sigma_{ij}^{(pq)}(\mathbf{x};\omega;t)=C_{ijkl}(\mathbf{x};\omega;t)\,\varepsilon_{kl}^{(pq)}(\mathbf{x};\omega;t) \\ C_{ijkl}(\mathbf{x};\omega;t)=\psi_1(\mathbf{x})\,C_{ijkl}^{(1)}(\omega;t)+\left(1-\psi_1(\mathbf{x})\right)C_{ijkl}^{(2)}(\omega;t) \end{cases}$$

where $\mathbf{x}\in\Omega$; i,j,k,l=1,2; p,q=1,2 (indices for 3 different homogenization problems); ω belongs to the admissible probabilistic space; $t\in[0,T]$; $\psi_1(\mathbf{x})$ is a characteristic function for the fiber region (equals 1 and 0 elsewhere); upper indices 1 and 2 at the constitutive tensors denotes the fiber and the matrix regions, respectively. The boundary conditions accompanying this problem are

$$\begin{cases} \chi_i^{(pq)}(\mathbf{x};\omega;t)=0,\ where\ \vec{n}_i\perp\Gamma,\ \mathbf{x}\in\Gamma \\ \dfrac{\partial\chi_j^{(pq)}(\mathbf{x};\omega;t)}{\partial x_i}=0,\ where\ \vec{n}_i\perp\Gamma,\ \mathbf{x}\in\Gamma \\ \sigma_{ij}^{(pq)}(\mathbf{x};\omega;t)\,n_j=\left|C_{pqij}(\mathbf{x};\omega;t)\right|n_j,\ \mathbf{x}\in\partial\Omega_{12} \end{cases}$$

where the last condition, containing a difference of the elasticity tensor for both components, holds true for the fiber-matrix interface. The constitutive tensor is defined here as

$$C_{ijkl}(\mathbf{x};\omega;t)=A_{ijkl}(\mathbf{x})\,e(\mathbf{x};\omega;t),$$

for the following stochastic representation of Young modulus in composite:

$$e(\mathbf{x};\omega;t)=-\dot{e}(\mathbf{x};\omega)\,t+e^0(\mathbf{x};\omega).$$

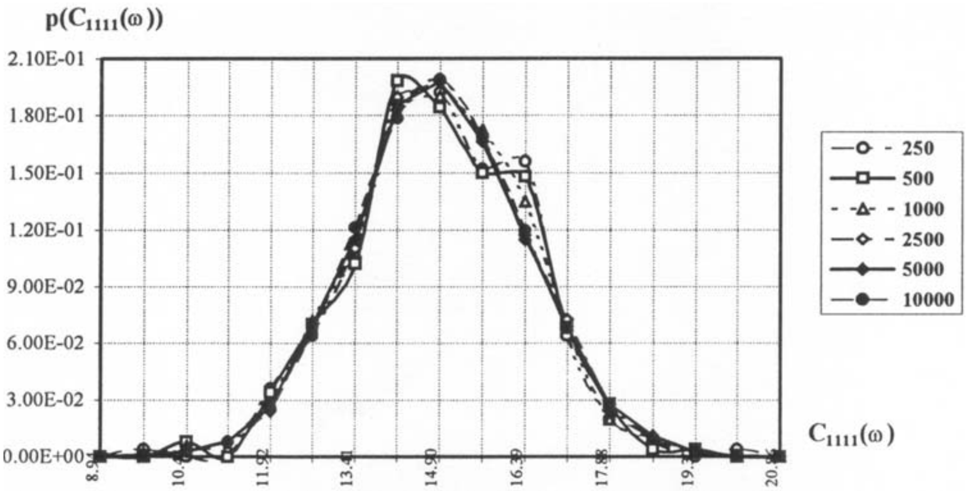
The random field $e^0(\mathbf{x};\omega)$ is equivalent to the initial elastic characteristics of the composite constituents, whereas $\dot{e}(\mathbf{x};\omega)$ represents the velocity of ageing process for the matrix and the fiber separately, i.e.

$$\dot{e}(\mathbf{x};\omega) = \psi_1(\mathbf{x})\,\dot{e}_1(\omega) + (1-\psi_1(\mathbf{x}))\,\dot{e}_2(\omega) \ .$$

All the aforementioned random quantities are truncated and uncorrelated Gaussian fields with the specified and finite first two moments. Let us note that such a representation of stochastic Young modulus means that it decreases monotonously for both composite components at the same time. It is apparent that this definition of the ageing process should result in a linear decrease of the expected values for the particular components of the effective elasticity tensor as well as in parabolic increase of the variances of those components. Then, the effective elasticity tensor may be defined as

$$C_{ijkl}^{(eff)}(\omega;t) = \left\langle C_{ijkl}(x;\omega;t)\right\rangle_{\Omega} + \left\langle \sigma_{ij}^{(kl)}(x;\omega;t)\right\rangle_{\Omega} .$$

Numerical simulation is provided here using the system MCCEFF [1] for the Monte-Carlo simulations of a homogenization problem solved using the Finite Element Method (the 4-noded plane strain finite element). Material parameters are adequate to the glass fibers embedded in epoxy matrix, where Young moduli have less than 20% random dispersion around their expectations. The ageing velocity is so adopted that it reflects the loss of 0.5% of the initial value for both composite components in each year.



A sample probability distribution function for the component process $C_{1111}^{(eff)}(\omega;t)$ at the beginning of the so defined ageing process is given above as a function of various numbers of the samples in the Monte-Carlo simulations. The fiber has round shape, is centrally located in the RVE and occupies 50% of its area; both components are perfectly connected here.

REFERENCES

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