Reflection and refraction of plane quasi-longitudinal waves at an interface of two piezoelectric media under initial stresses.

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Abstract:

This paper is devoted to study a problem of reflection and refraction of quasi-longitudinal waves under initial stresses at an interface of two anisotropic piezoelectric media with different properties. One of the two media is Aluminum Nitride (ALN) which is considered the down piezoelectric medium and the above medium is chosen as PZT-5H ceramics. The two piezoelectric media in welded are assumed to be anisotropic of a type of a transversely isotropic crystals (hexagonal crystal structure, class 6 mm). The equations of motion and constitutive relations for the piezoelectric media have been written. Suitable mixed boundary conditions are used at the interface of the two piezoelectric media. It has been shown analytically that both reflected and refracted *quasi-longitudinal waves (qP)* and *quasi- shear vertical waves (qSV)* depend upon material constants of the two media, the initial stresses, the constants of electric potential and the angle of incidence. The numerical values of the reflection and refraction coefficients of different transversely isotropic media at different angles of incidence longitudinal plane waves were obtained and plotted. Some special cases are considered and the values were also compared to the corresponding values of anisotropic media without the electric effects and in the absence of effects of the initial stresses. It is found that these coefficients reflection and refraction are functions of angle of incident, elastic constants, piezoelectric potential parameters and the initial stresses. Numerical computations and the results obtained are depicted graphically. Finally, some particular cases have also been reduced from the present study. This investigation is considered important due to the initial stresses in the such practical problems are inevitable and important because they may result in frequency shift, a change in the velocity of surface waves and controlling the selectivity of a filter compensation of the devices.

1. Basic equations and formulation of the problem [2], [3].

We will consider a transversely isotropic piezoelectric half-space occupying region $z \le 0$ and adjoining the vacuum $z > 0$. Let the wave motion in this medium be characterized by: the displacement vector $\vec{u}(u,0,w)$, the electric potential function φ, all these quantities being dependent only on the variables x, z, t.

The general forms of the dynamical equations of motion with homogeneous initial stresses and the equation for electric field can be expressed as follows:

$$\sigma_{ij,i} + u_{j,ki}\overset{\circ}{\sigma}_{ik} = \rho \ddot{u}_j, \quad D_{i,i} = 0. \tag{1}$$

Completed by the constitutive equations:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \quad D_i = e_{ikl}\varepsilon_{kl} + \in_{ik} E_k \tag{2}$$

The relationship between the displacement components and strain components and the Maxwell equations in the quasi-static approximation are given by

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_k = -\phi_{,k}. \tag{3}$$

where σ_{ij}, ε_{kl}, E_k and D_i indicate the stress tensor, strain tensor, electric field vector, and electric displacement vector, respectively, are c_{ijkl}, e_{kij} and $\in_{ik}$ are elastic, piezoelectric and dielectric constants for piezoelectric medium, with $i, j, k, l = 1, 2, 3$ and ρ is the mass density, u_i denote the mechanical displacements in the ith direction, σ°_{ik} is the initial stress tensor. The dot denotes time differentiation, the comma denotes space-coordinate differentiation, the repeated index in the subscript implies summation.

2. Solution of the problem for incident *qP*-waves and boundary conditions [1]:

Let us consider the solution of Eqs. $(1)_1$ and $(3)_2$ as:

$$(\vec{u}^{(n)}, \varphi^{(n)}) = (\vec{A}, \vec{B})\, \vec{d}_n \exp(i\eta_n), \quad \text{with} \quad \eta_n = K_n(\vec{x} \cdot \vec{p} - c_n t) \tag{4}$$

where different values of the index n serve to label the various types of waves that occur, $\vec{d}$ is the unit displacement vector, $\vec{p}$ being the unit propagation vector, $\vec{x}$ is position vector, with $\vec{x} \cdot \vec{p} = \text{const.}$ is plane of constant phase, $\vec{A}, \vec{B}$ are the vibration amplitudes, c_n the velocity of propagation and K_n the corresponding wave number.

The boundary conditions at $z = 0$ may be considered as:

$$w^{(\circ)} + w^{(1)} + w^{(2)} = w^{(3)} + w^{(4)}, \quad \sigma^{(\circ)}_{zx} + \sigma^{(1)}_{zx} + \sigma^{(2)}_{zx} = \sigma^{(3)}_{zx} + \sigma^{(4)}_{zx},$$
$$\sigma^{(\circ)}_{zz} + \sigma^{(1)}_{zz} + \sigma^{(2)}_{zz} = \sigma^{(3)}_{zz} + \sigma^{(4)}_{zz}, \quad \varphi^{(\circ)} + \varphi^{(1)} + \varphi^{(2)} = \varphi^{(3)} + \varphi^{(4)}. \tag{5}$$

Substituting in (5), the values of $\sigma^{(n)}_{zz}, \sigma^{(n)}_{zx}$ and $\varphi^{(n)}$ (for $n = 0,1,2,3,4$) from (1)-(3) using Eqs. (4), one may have the reflection and refraction coefficients of different transversely isotropic media at different angles of incidence longitudinal plane waves. The two different materials will be considered ceramic PZT-4, Aluminum Nitride (AlN). The numerical values of reflection and refraction coefficients for different initial stresses and the angle of incidence have been calculated by computer and the results are given in the form of graphs.

3. References:

[1] Achenbach J.D., "Wave propagation in elastic solids", North-Holland Publishing Company-Amsterdam, Vol. 16, (1973).

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[3] Dieulesaint E. and Royer D., "Elastic waves in solids", John Wiley & Sons, New York, (1980).