8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Convergence of a time-stepping scheme for multibody dynamics with unilateral constraints

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Key Words: *Dynamics with perfect unilateral constraints, Moreau's impact law, time-stepping scheme*

ABSTRACT

Let us consider a system of rigid bodies submitted to perfect unilateral constraints. More precisely, let us denote by $q \in \mathbb{R}^d$ the representative point of the system in generalized coordinates. The unconstraint dynamics is described by the following ODE

$$M(q)\ddot{q} = g(t, q, \dot{q}) \tag{1}$$

where $M(q)$ is the mass matrix. We assume that the system is submitted to frictionless unilateral constraints, i.e. the configurations $q(t)$ should remain in a set K given by

$$q(t) \in K = \left\{ q \in \mathbb{R}^d; f_\alpha(q) \geq 0 \quad \forall \alpha \in \{1, \dots, \nu\} \right\}, \quad \nu \geq 1. \tag{2}$$

The inequalities $f_\alpha(q) \geq 0$ describe non-penetrability conditions and when $q(t) \in \partial K$, i.e. when there is contact, we assume that the transmission of the velocity is given by Moreau's impact law

$$\dot{q}(t+0) = \mathrm{Proj}_{q(t)}\big(T_K(q(t)), \dot{q}(t-0)\big) \tag{3}$$

where $\mathrm{Proj}_q(T_K(q), \cdot)$ denotes the projection on the tangential cone to K at q relatively to the kinetic metric at q (see [8]).

For this problem we adopt the velocity-based formulation as a measure-differential inclusion proposed by J.J. Moreau, so we replace (1)-(2)-(3) by

$$g(t, q, u)\, dt - M(q)\, du \in N_{T_K(q)}(u) \tag{4}$$

where $u = \dot{q}^+$ and $N_{T_K(q)}(u)$ is the normal cone to $T_K(q)$ at u. Then, starting from (4), we consider the following time-discretization: for all $i \geq 0$

$$\begin{cases} q^{i+1} = q^i + hu^i \\ g(t_{i+1}, q^{i+1}, u^i) - M(q^{i+1}) \left(\dfrac{u^{i+1} - u^i}{h} \right) \in N_{T_K(q^{i+1})}(u^{i+1}) \end{cases}$$

with the appropriate initialization $q^0 = q_0$, $u^0 = u_0$, where $(q_0, u_0) \in K \times T_K(q_0)$ are given admissible inital data.

This scheme has been proposed by J.J. Moreau (see [8] or the review paper [6]) and has been successfully implemented to solve numerically problems related to granular dynamics (see [9]) or automotive industry (see [2], [3]).

In the case of a single constraint, i.e. $\nu = 1$, the convergence of this scheme has already been established (see [7] when $M(q) = \mathrm{Id}_{\mathbb{R}^d}$ and [4], [5] when the mass matrix is not trivial) but the assumption $\nu = 1$ is quite restrictive and does not suit well for multibody dynamics where in general $\nu \geq 2$. Hence it is important to treat also the multi-contraint case but we meet a new difficulty, due to the lack of continuity with respect to data. Nevertheless, following [1] and [10], we know that continuity on data holds if the active constraints create right or acute angles and, in this framework, we prove once again the convergence of the time-stepping scheme.

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