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Maximum likelihood estimation and polynomial chaos expansions in stochastic inverse problems

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ABSTRACT

Uncertainty quantification in computational mechanics has received much attention in the past few years. Based on a mathematical model of a system and a probabilistic description of its input parameters, the response joint probability density function is usually of interest, as well as derived quantities such as statistical moments or the probability of exceeding some threshold (reliability analysis).

In this context, accurate predictions require that both the mechanical model and the probabilistic model of its input parameters reflect the real world. In many situations, there is few data available for the probabilistic modelling, if any. However, there are cases where the uncertainty in the input may be grasped through the observation of the scattering of the response of various identical systems (e.g. within an experimental program).

In this communication a framework is proposed to tackle these so-called *stochastic inverse problems*. Based on a model of a system $Y = \mathcal{M}(\boldsymbol{X}_1, \boldsymbol{X}_2)$ (where the PDF of input vector $\boldsymbol{X}_2$ is prescribed) and a set of observations of identical system responses, say $\left\{y^{(1)}, \cdots, y^{(Q)}\right\}$ (possibly affected by measurement error), the PDF of $\boldsymbol{X}_1$ is to be estimated.

As the type of distribution of $\boldsymbol{X}_1$ is not known in advance, and in order to avoid additional assumptions on its belonging to a class of distributions, a semi-parametric representation of $\boldsymbol{X}_1$ is used. For this purpose, polynomial chaos (PC) expansions [1, 2, 3] are introduced, which allow one to represent intrinsically $\boldsymbol{X}_1$ in terms of a set of coefficients: $\boldsymbol{X}_1 = \sum_{j \geq 0} a_j \; \Psi_j(\boldsymbol{\xi})$, where $\{\Psi_j(\boldsymbol{\xi}) \;,\; j \geq 0\}$ is e.g. the Hermite polynomial chaos basis.

The coefficients a_j's are computed by maximum likelihood estimation. In order to compute explicitly the PDF of $\boldsymbol{X}_1$ (conditionally to the values of a_j's), kernel smoothing techniques are used [4].

In order to assess the accuracy of the technique, the approach is applied to the identification of elastic properties of simple structures.

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