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RATE-INDEPENDENT ENERGETIC APPROACH TO STOCHASTIC DAMAGE MECHANICS OF DISCRETE LATTICES

***Jan Zeman[1], Ron H.J. Peerlings[2] and Marc G.D. Geers[3]**

[1]Department of Mechanics, Faculty of Civil Engineering, CTU in Prague
Thákurova 7, 166 28 Praha 6
zemanj@cml.fsv.cvut.cz

[2]Materials Technology, Mechanical Engineering, TU/e
PO Box 513, WH 4.135 5600 MB, Eindhoven
R.H.J.Peerlings@tue.nl

[3]Materials Technology, Mechanical Engineering, TU/e
PO Box 513, WH 4.135 5600 MB, Eindhoven
M.G.D.Geers@tue.nl

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ABSTRACT

Random lattice models of damage represent a heterogeneous material as discrete units interacting via brittle elements. By randomizing the individual links' properties and repeated Monte-Carlo analyses, such models are able to naturally incorporate a variety of phenomena, which are inherently difficult to capture by convectional continuum theories. Most importantly, the discrete models are well-suited to resolve the multi-scale character of damage evolution without the need of the separation-of-scale hypothesis, cf. [1]. Even though the discrete lattice models provide a convenient algorithmic approach to damage simulation for given material data and loading program, they fail to provide an insight into the governing equations of the non-local collective behavior. In this contribution, we outline an approach to a rational derivation of global stochastic damage theories for finite discrete lattices.

A unifying theoretical approach to the analysis builds on recent advances in the theory of rate independent processes [3], describing evolution of a system as a competition between the globally stored elastic energy $E(t, \boldsymbol{d}, \boldsymbol{\chi})$ and the global dissipation $D(\boldsymbol{\chi}^1, \boldsymbol{\chi}^2)$ (with $\boldsymbol{d}$ storing the nodal displacements and the binary vector $\boldsymbol{\chi}$ describing the state of each element in the structure, cf. [2]). In this framework, the discrete damage evolution is equivalent to an incremental evolution of internal variables, describing changes in the structure topology due to the spatial distribution of failed trusses. Formally, the damage process can be represented as a time incremental problem:

$$\begin{aligned} \boldsymbol{\chi}(t_{i+1}) &= \arg\min_{\boldsymbol{\chi}\in\mathbb{S}} H(t_{i+1}, \boldsymbol{\chi}) + D(\boldsymbol{\chi}(t_i), \boldsymbol{\chi}) \\ \text{with } H(t_{i+1}, \boldsymbol{\chi}) &:= \min_{\boldsymbol{d}\in\mathbb{K}(t_{i+1})} E(t_{i+1}, \boldsymbol{d}, \boldsymbol{\chi}) \end{aligned} \tag{1}$$

where $\mathbb{K}$ denotes the set of kinematically admissible displacements the and $\mathbb{S}$ is the set of internal variables.

When extended to the stochastic format, the description of failed elements is possible only in terms of appropriate *statistics* of internal variables, which now constitute the generalized internal variables. In

order to account for binary interactions between units, we introduce the two-element probability matrix $\boldsymbol{P}$, storing the probabilities of simultaneous failure of two elements in the structure.

Such a step requires an appropriate redefinition of both components of the deterministic model. First, the generalized stationary energy $\mathcal{H}(t_{i+1}, \boldsymbol{\chi})$ can derived using recent ideas from the field of random composites [4]. The treatment of the generalized global dissipation is, however, substantially more involved $D(\boldsymbol{P}^1, \boldsymbol{P}^2)$ and only approximation in terms of the local (first-order) statistics is available, see [5] for more details. This yields the incremental generalization of (1)

$$\boldsymbol{P}(t_{i+1}) = \underset{\boldsymbol{P}\in\mathcal{S}}{\arg\min}\, \mathcal{H}(t_{i+1}, \boldsymbol{P}) + \mathcal{D}(\boldsymbol{P}(t_i), \boldsymbol{P}), \tag{2}$$

which is a convex program efficiently solvable using modern optimization tools.

The accuracy of the developed description is illustrated on a simple structure shown in Figure 1 by comparing the estimated energies with results of a simple Monte-Carlo analysis. Clearly, while stored energy estimate shows a reasonable match (especially when compared to the first-order bound), the first-order energy dissipation formula underestimates the dissipated energy. This simple example shows that not only non-local expression for the stored energy, but also a non-local dissipative mechanics needs to be correctly accounted for to arrive at a truly predictive model.

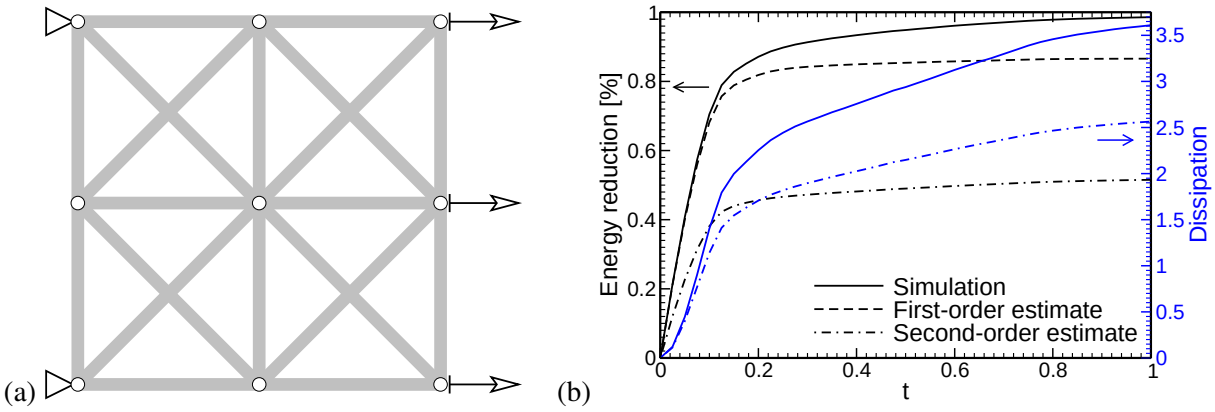

Figure 1: (a) Scheme of example, (b) Predicted vs simulated energetics

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