Multi-level parametric shape optimization for reflector antennas design

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ABSTRACT

A reflector antenna is a system composed of a primary electromagnetic wave source (waveguide, dipole) and a diffracting obstacle (reflector). We aim to optimize the geometry of the reflector w.r.t. some physically relevant properties of such antennas given by functionals of the far field radiation pattern. Two specific applications are considered: a power synthesis test-case (inverse problem) and a directivity uniformisation problem. The system is assumed to be axisymmetric (Figure 1). The Maxwell equations solver SRSR provided by France Télécom R&D La Turbie [1] has been used to compute the far field.

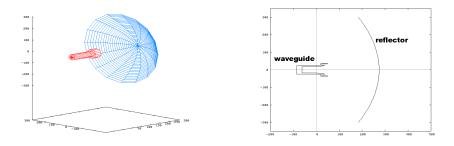


Figure 1: 3D and 2D representations of the axisymmetric system.

From the numerical point of view, the shape of the reflector is represented in a finite-dimensional space (shape parameterization). The control variables, or design parameters, are optimized with a quasi-Newton method using the BFGS formula for the update of the quasi-Hessian. In this framework, the question of achieving a high accuracy is still an important and difficult issue. The accuracy of the final numerical result is influenced by both the geometrical representation (finite-dimensional approximation) and the iterative algorithm performance (quality of partial convergence). Indeed, the larger the search space is, the more accurately the optimal geometry is approximated but the slower the algorithm converges due to numerical stiffness, and full convergence more difficult to achieve.

We focus on two numerical issues: first, the choice of the shape parameterization is discussed, i.e. how should the finite-dimensional search space and its basis be chosen; then we discuss how can recent developments in multi-level optimization algorithms be applied to improve the convergence rate [2,3].

A natural choice for the shape representation is to use the Bernstein polynomials (Bézier representation), as commonly done in the Computer Aided Design (CAD) field. However, a numerical experimentation shows that this basis is not well adapted: a better convergence rate is obtained with Tchebychev polynomials (Figure 2). In the case of the power synthesis problem, for which the exact solution is known, a study of the Hessian matrix at convergence is conducted. It appears that the condition number of the Hessian matrix is far smaller when using the Tchebychev basis, indicating a much better conditioned system.

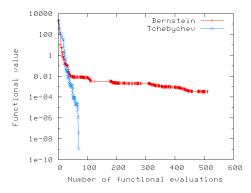


Figure 2: Convergence history of optimization iteration for the Bernstein and Tchebychev polynomial basis (power synthesis problem).

Secondly, we focus on multi-level algorithms. In order to illustrate the need for such algorithms, a spectral analysis of the Hessian for some shapes exhibits the modes that are slow to converge: high frequency modes corresponding to small eigenvalues (Figure 3). Based on this, different strategies, inspired from multigrid methods, have been constructed using embedded search spaces for the geometrical representation defined by the eigenvector structure of the Hessian matrix. Improvements in convergence rate by factors from 2 to 10, depending on the test case, have been measured by multi-level approaches.

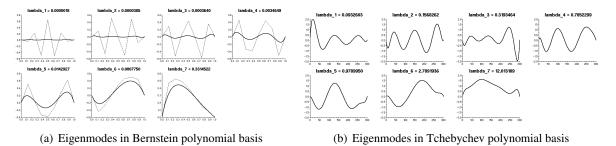


Figure 3: Eigenmodes, or eigenvectors of the Hessian matrix of the criterion w.r.t the design parameters (power synthesis problem).

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