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Direct versus iterative solvers for sparse linear systems within time-adaptive finite-element analysis of inelastic structures

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ABSTRACT

Many engineering applications involving inelastic structures lead to nonlinear initial boundary value problems, which are given as partial differential equations. Within this context, the material behavior can be described by constitutive models of evolutionary type. The materials state is given by so called internal variables, which develop according to either ordinary differential equations or differential-algebraic equations. Examples for internal variables are plastic strains and backstresses describing kinematic hardening behavior.

The result of the spatial discretization of the given initial boundary value problem using finite elements can be interpreted as a system of differential-algebraic equations, [3]. They can be solved efficiently by time-adaptive implicit time integration methods. Here, diagonally implicit Runge-Kutta methods are used. Every time step requires the solution of one or more non-linear equation systems, depending on the order of the chosen integration method. Applying the Multilevel-Newton algorithm within this approach preserves the classical iteration scheme of local and global iterations and leads to a sequence of large sparse linear systems, see [4]. Typical numbers of required solutions vary between three and thirty for every single time step. The solution process of these systems contributes a major portion to the total computational costs and is strongly affected by the properties of the coefficient matrices (tangential stiffness matrices).

According to the chosen type of material model and the spatial discretization the matrices become large, non-symmetric and ill-conditioned. Due to these properties some solvers are more efficient than others and some require preconditioning. In our contribution we study the influence of two different material models. The first one describes metal viscoplasticity for large deformations, [6,11]. It is based on a yield function of von Mises-type and two multiplicative decompositions of the deformation gradient, which define two inelastic intermediate configurations. These configurations develop during plastic loading according to the underlying evolution equations. The model consists of a total of twelve independent internal variables and is highly nonlinear. The second model describes the plastic behavior of a polymer

(polyoxymethylene) within the small strain regime, [5]. This model represents a smooth but non-linear problem.

In our study we investigate the performance of direct and iterative solvers on shared memory multiprocessor machines, which have become mainstream computers. As sparse direct solvers we are using the current version of UMFPACK, [2], and the parallel version of PARDISO, [9]. The latter is run with one to eight threads. The results are compared to the performance of preconditioned Krylov-subspace solvers as GMRES, FGMRES and BiCGStab. The applied preconditioners are based on incomplete LU factorization techniques as ILU0 and ILUT, [8]. For this combination the iterative solvers outperform the direct ones, provided that sufficiently large models are used. It will be shown that in the case of iterative solvers the tolerance of the solver can be adjusted to the current error within the Newton scheme, [7]. This adaption yields a performance increase. A further gain is achieved by using a preconditioner update-technique. The Newton-type iteration leads to a sequence of gradually changing values of the nonzero coefficient matrix, but the same sparsity pattern. Thus, the preconditioner can be determined at the beginning of the solution process and be updated only when needed. The novel preconditioner update-technique for nonsymmetric systems, [1,10], offers an additional and significant speedup.

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