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Smoothness-Increasing Accuracy-Conserving filtering of discontinuous Galerkin solutions

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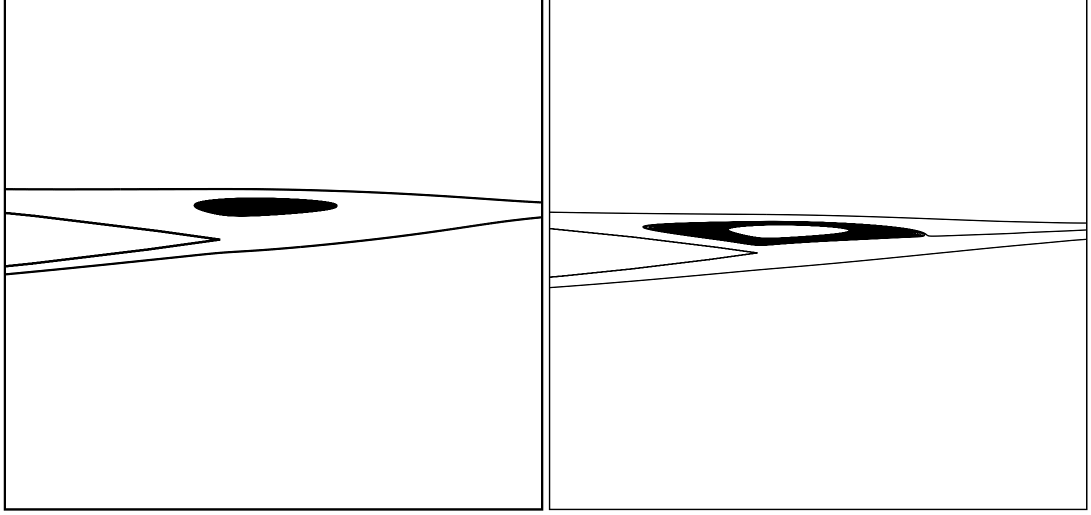
ABSTRACT

The discontinuous Galerkin (DG) method continues to be a popular method due to several nice features including flexibility for adaptivity and ability to maintain high order approximations when discontinuities occur at inter-element boundaries. These nice properties can lead to complications when using the DG approximation for feature extraction and visualization. There has been previous investigations into post-processing of discontinuous Galerkin solutions for linear hyperbolic equations [1]. In this instance, the post-processor increases the accuracy of the DG solution over quadrilateral meshes from $\mathcal{O}(h^{k+1})$ to $\mathcal{O}(h^{2k+1})$ while filtering out oscillations in the error. This technique seems promising for filtering in visualization applications, specifically streamline integration.

There are two alternatives for filtering the discontinuous Galerkin method for use in streamline integration. This smoothness-increasing accuracy-conserving (SIAC) filtering technique enhances the smoothness of the field and eliminates discontinuities between elements, thus resulting in more accurate streamlines. The first implementation involves filtering the entire field, which aids in the ability to use a less restrictive time integrator [2]. For multi-dimensional data, this filtering is done in a tensor-product way. The second alternative for streamline integration involves using a one-sided post-processing kernel together with a suitably chosen characteristic length for filtering multi-dimensional data in a one-dimensional manner [3]. By implementing it in a one-dimensional manner along a streamline, the computational cost is reduced. It is this second method that will be the focus of this talk.

For example, a two dimensional viscous flow is calculated around a NACA 0012 airfoil with $\alpha = 1.25$, $M = 0.5$ and $Re = 5 * 10^{-3}$. We are interested in obtaining the streamline close the body, where separation can be observed toward the trailing edge of the airfoil.

In Figure 1, a close up of the streamline in the separated region at the trailing edge is shown. The non-filtered streamline (left) spirals inward toward a critical point, which is a physically incorrect result.



(a) Non-filtered (b) filtered

Figure 1: Streamlines at the separated region behind a NACA 0012 airfoil. The streamline starting location is the same for both cases. $\Delta t = .1$

However, by implementing the smoothness-increasing accuracy-conserving filter, we are able to obtain a more correct streamline that spirals out and finally leaves the airfoil (right image).

This example demonstrates how, when using ODE integrators, errors can accumulate and produce erroneous results especially in the presence of critical points. The results demonstrate that this post-processor is an effective means of eliminating incorrect integration of non-physical DG solutions close to critical points and that we are able to obtain more accurate streamlines using larger timesteps.

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