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Variational Germano for stabilized Stokes and Oseen flow

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ABSTRACT

It is well known that standard Galerkin methods often need to be augmented with stabilizing terms. In the realm of fluid problems, this necessity can usually be traced back to either the mixed nature of these problems or dominating convection. Both issues have already been resolved in the 80s resulting in a.o. [1][2]. Physical justification of these stabilized methods came with the introduction of the variational multiscale (VMS) method [3].

When the variational multiscale method is consistently applied to the Navier-Stokes equations the classical stabilization terms (SUPG/PSPG/LSIC) are recovered. However, due to the nonlinear convection additional terms appear. Together with the SUPG term these can be interpreted as a Large-eddy simulation (LES) turbulence model [4][5].

Much effort has been put in designing stabilization parameters. Emphasis has been on optimality in terms of the *order* of accuracy. However, this is of secondary importance in the case of LES, as we are trying to converge to a solution with fractal character ($E(k) \sim k^{-\frac{3}{5}}$). In the case of LES absolute accuracy is of greater importance. Hence in the definition of the stabilization parameter, it is not sufficient to consider only the scaling with respect to mesh size in a certain limit. The coefficient of proportionality as well as the transition from one limit to another is of importance.

To chose an *optimal* stabilization parameter the use of Variational Germano Identity (VGI) is investigated, as recently introduced in [6]. In this setting *optimal* means a stabilization parameter which results in a discrete solution (u^h) which is a specified projection of the exact solution (u). In this sense VGI is a natural extension of VMS. We consider Stokes and Oseen flow to test the validity and robustness of these method.

REFERENCES

[1] A.N.Brooks and T.J.R.Hughes. "Streamline Upwind/Petrov-Galerkin formulations for convection dominated flows with particular emphasis on the incompressible Navier-Stokes equations'. *Comput. methods Appl. Mech. Engrg*,Vol. **32**, 199–259, 1982.

[2] T.J.R.Hughes,L.P.Franca and M.Balestra. "A new finite element formulation for computational fluid dynamics: V Circumventing the Babuska-Brezzi condition: A stable Petrov-Galerkin formulation of the Stokes problem accomodating equal-order interpolations". *Comput. methods Appl. Mech. Engrg*,Vol. **59**, 85–99, 1985.

[3] T.J.R. Hughes, G.R. Feijóo, L. Mazzei and J.B. Quincy. "The variational multiscale method – a paradigm for computational mechanics". *Comput. methods Appl. Mech. Engrg*,Vol. **166**, Issues 1-2, 3–24, 1992.

[4] Y. Bazilevs, V.M. Calo, J.A. Cottrell, T.J.R. Hughes, A. Reali and G. Scovazzic. "Variational multiscale residual-based turbulence modeling for large eddy simulation of incompressible flows". *Comput. methods Appl. Mech. Engrg*, Vol. **197**, Issues 1-4, 173–201, 2007.

[5] I. Akkerman, Y. Bazilevs , V.M. Calo, T.J.R. Hughes and S.J. Hulshoff. "The role of continuity in residual-based variational multiscale modeling of turbulence". *Computational Mechanics*, to appear.

[6] A.A. Oberai and J. Wanderer. "A dynamic approach for evaluating parameters in a numerical method". *Internat. J. Numer. Methods Engrg*,Vol. **62**, 50–71, 2005.