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A PRECONDITIONED SECOND-ORDER LINEARIZED IMPLICIT FORMULATION FOR BAROTROPIC CAVITATING FLOWS

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ABSTRACT

In the present study, the improvement of a numerical frame aimed to simulating 3D cavitating flows is presented, the final goal being to be able to simulate propellant flows occurring in the feed turbo-pumps of modern liquid propellant rocket engines. As for the cavitation model, a homogeneous-flow approach, accounting for thermal effects and active nuclei concentration (see [1]), is considered, which leads to a barotropic state law. The 3D continuity and momentum equations for compressible inviscid flows are discretized by a finite-volume approach, applicable to unstructured grids, and in which the numerical fluxes are evaluated by an ad-hoc developed adaptation of the Roe-type upwind scheme, suitable for a generic barotropic fluid. Furthermore, time advancing is carried out by an implicit linearized scheme, in which the linearization only exploits the properties of the Roe matrix.

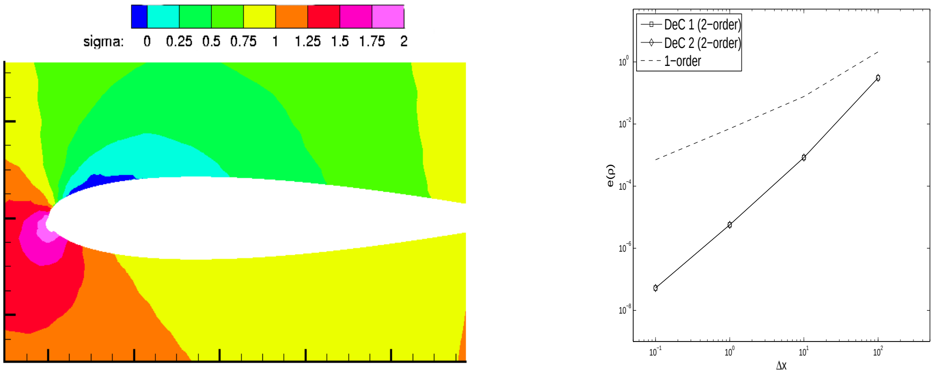
Note that taking into account cavitation phenomena in such a model appears as a very difficult task from a numerical viewpoint. Indeed, the local presence of both incompressible zones (pure liquid) and regions where the flow may become highly supersonic (cavitating mixture) renders the problem particularly stiff. First of all, compressible solvers exhibit efficiency problems as well as accuracy problems at low Mach number (i.e. when dealing with nearly-incompressible flows). A preconditioning scheme, similar to the one proposed in [2] for the case of a perfect-gas, was defined for barotropic flows to overcome the difficulties related to the accuracy in the low Mach number limit, while the implicit formulation should cure the efficiency problems. The whole formulation, described more in details in [3], was tested on 1D validation benchmarks, 3D cavitating flows around a hydrofoil [4] (see e.g. Fig. 1a) and 3D rotating (but non-cavitating) flows in a turbo-pump inducer [4, 5].

Then, the extension to second-order accuracy (both in time and space) at a reduced computational cost was achieved through a classical MUSCL spatial reconstruction associated to a defect-correction formulation [6]. This recent study was focused in a 1D context within which it is possible to define exact benchmark solutions (Riemann problems, quasi 1D convergent-divergent nozzle). The validation (see e.g. Fig 1b) was carried on a large variety of 1D test-cases involving different flow regimes (from low

Mach number to supersonic flows considering both steady and unsteady problems) as well as different degrees of regularity of the solution (smooth, continuous and discontinuous). Since second-order appears as the least order of accuracy required for practical applications, the implementation and validation of the formulation proposed in [6] for 3D cases is the first objective of the present work.

Improvements in the flow modeling, and in particular, a study on the viscous effects, neglected at a preliminary stage, will also be presented.

Although the numerical formulation proposed in [3] can deal in principle with 3D barotropic flows in low Mach number regime, when cavitating phenomena are present in the flow, the simulations are in practice extremely difficult for real industrial problems (as, e.g. the flow in a turbo-pump inducer). Indeed, even with an implicit formulation, very severe restrictions on the time-step are needed for stability and this leads to prohibitive computational costs. Thus, investigations, in a first stage in a 1D context, will be provided in the present study to address and counteract this main difficulty. They will mainly be focused on the improvement in the definition of the preconditioning, based on an asymptotic analysis in power of the Mach number for both continuous and discrete equations, as well as on the use of relaxation techniques and time-stepping strategies.



(a) (b)

Figure 1: (a) Cavitating flow on hydrofoil (from [4]): contour plot of the local cavitation number; (b) supersonic flow in a 1D nozzle: error behavior for first- and second-order approaches.

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