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Robust and direct evaluation of J_2 in linear elastic fracture mechanics with the X-FEM

*** G. Legrain[1], N. Moës[1] and E. Verron[1]**

[1] GeM Institute
École Centrale de Nantes,
1 Rue de la Noë
BP92101
44321 Nantes, France.
{gregory.legrain,
nicolas.moes,
erwan.verron}@ec-nantes.fr

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ABSTRACT

Numerical simulation of fracture mechanics is of major importance in the design of structures such as planes or dams. For such complex structures, robustness and accuracy of the simulations are necessary. The finite element method is classically considered for the simulation of such problems. However, the crack must be explicitly meshed in order to incorporate the discontinuity of the displacement field across crack faces. This process has to be handled at each step of propagation. The main drawback of this approach is the computational and human cost of remeshing. The X-FEM method was proposed as a response to this issue [1] [2]. It uses the concept of Partition of Unity [3] in two ways : first to take into account the displacement jump across crack faces far away from the crack tip, and also to enrich the approximation close to the tip with some analytical asymptotic fields. Another important issue in fracture mechanics is the prediction of crack growth direction. Several criteria have been proposed: one can cite for example the maximum hoop stress criterion [4], the maximum tangential stress criterion or the symmetry principle [5]. The maximum energy release rate criterion [6], also called "vectorial J-integral" criterion [7] is an alternative which was initially proposed by Hellen and Blackburn [7]. The main difference in evaluating this criterion with respect to the classical J-integral rely on the difficulties in the evaluation of the second component J_2 of the energy release rate vector $\mathbf{J}$. It is well known that J_1 (which is basically the J-integral) is path independent. Concerning J_2, Herrmann and Herrmann [8] have shown that it is path independent only in a modified manner, i.e. by adding an extra contribution on the crack faces. However, this new contribution requires an accurate determination of the mechanical fields near the crack-tip. It is well known that the quality of these fields near a singularity is not ensured in the finite element context. This is why some authors proposed specific strategies to avoid the integration near the tip for an accurate numerical evaluation of J_2 [9, 10]. For enriched finite

element methods, the only study on this subject was recently proposed by Heintz [11] in the context of configurational forces. His approach is directly related to the one proposed in [10], except that the near-tip integration is not avoided. The major advantages of this direct approach are that it does not necessitate near-tip fields assumptions, and that it requires only one computation of domain integral.

Nevertheless, no study has been proposed to evauate the accuracy of the above mentioned methods. The aim of the present paper is to compare various numerical approaches for the evaluation of the $\mathbf{J}$-integral. It is shown that direct approaches do not converge at their optimal rate, but that an optimal convergence can be recovered by mean of an improved strategy which is proposed under the hypothesis of linear elastic fracture mechanics. Finally, examples of crack propagation using this criterion are presented, and the improvement of the robustness of the crack propagation process is illustrated.

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