## COVARIANT DESCRIPTION FOR CONTACT BETWEEN ARBITRARY CURVES: GENERAL APPROACH FOR BEAMS, CABLES AND SURFACE EDGES

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## ABSTRACT

Recent developments [1], [2] in numerical contact mechanics especially in the case of large deformations have shown the high robustness of the covariant approach. Following the geometrical interpretation of this approach, contact interaction between bodies is considered as an interaction between surfaces and is carried out in a specially defined local surface coordinate surface related to the surface. This coordinate system is defined after the closest point projection (CPP) procedure in which a chosen "slave" point from one contacting body is projected to the "master" surface of another contacting body. For numerical algorithms all necessary parameters are related then to the surface coordinate system and are written in a covariant form. It leads to a description which is independent of the discretization of the contact surfaces e.g. by FE and can be applied even for high-order FE with exact representation of the geometry [3].

The geometrical approach for curve-to-curve interaction appears to be more complicated because both contacting points should be found via the CPP procedure [4]. Only a few publications are known mostly for cases with linear geometry; e.g. a case with curvilinear beams is considered in [5] in which all necessary derivatives have been computed via the symbolic algebra program.

Characterizing the geometrical diversity of contact situations the beam-to-beam contact is not only a case with curve-to-curve interactions. Such geometrical situations occur when bodies are interacting along their surface edges and e.g. cable contact etc. Thus, the aim of the current contribution is to develop such a general description containing only the necessary kinematical characteristics from the geometry of contacting curves as well as their necessary mechanical characteristics such as contact forces, sliding paths etc. and others.

The covariant approach starts with a definition of a contact point via the CPP procedure defined via arc-length parameters  $s_i$ :

$$||\boldsymbol{\rho}_1(s_1) - \boldsymbol{\rho}_2(s_2)|| \to \min, \quad \longrightarrow \begin{cases} (\boldsymbol{\rho}_1(s_1) - \boldsymbol{\rho}_2(s_2)) \cdot \boldsymbol{\tau}_1 &= 0\\ -(\boldsymbol{\rho}_1(s_1) - \boldsymbol{\rho}_2(s_2)) \cdot \boldsymbol{\tau}_2 &= 0 \end{cases}$$
(1)

The CPP procedures, in due course, result in a pair of Serret-Frenet frames  $\tau_i$ ,  $\nu_i$ ,  $\beta_i$  attached to both curves, see Fig. 1.

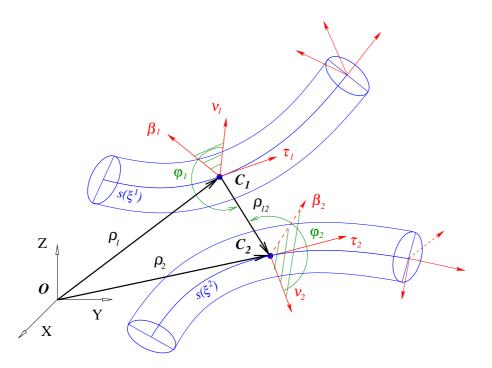


Figure 1: Serret-Frenet local coordinate frames attached to both curved lines. Definition of the closest point projection (CPP) procedure.

A further description is then given in the defined coordinate system and and follows the stages: a) study of the solvability conditions (existence, uniqueness) of the related CPP procedure; b) contact kinematics of curve-to-curve contact with consideration of independent relative motion of the selected curve; c) formulation of various constitutive relations for curve-to-curve contact; d) enforcement of contact conditions (Lagrange multiplier method, penalty method). All necessary contact terms for the further iterative solution as well as the necessary linearization are considered in the local coordinate systems attached to the curves. It leads to a formulation which is independent of the approximation of both curves. This description can be then independently applied for any case with curve-to-curve kinematics such as contact between curved beams and cables as well as contact between surface edges.

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