

Discontinuous Galerkin Methods for Miscible Displacement Problems

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ABSTRACT

Miscible displacement methods are increasingly used in oil recovery in order to enhance the production. For instance, currently about 4% of the US total is recovered by means of CO₂-based techniques.

In this talk a mathematical model is presented which models incompressible, miscible displacement of one fluid by another in a porous medium. The discretisation in the spatial variables consists of a discontinuous Galerkin method, which is combined with a mixed finite element method. For the time variable the implicit Euler method is used.

Under minimal assumptions on the regularity of the problem, it is shown that subsequences of numerical solutions converge to weak solutions of the problem. Novel features of the method are:

- (i) The construction of a scheme which does not require stabilisation or regularisation of the advection term.
- (ii) The proof of convergence does neither rely on the uniqueness of the exact solutions nor on convexity or smoothness of the domain.
- (iii) The dG approximation spaces are embedded to facilitate an analysis which is partially in form of a conforming method.