

Modified Augmented Lagrangian Methods for the Navier-Stokes Equations

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ABSTRACT

Augmented Lagrangian-based preconditioners have been shown to be highly effective and robust in the solution of the discrete Stokes and Oseen problem and in solving the linear systems arising from the stability analysis of (Newton) linearized solutions to incompressible flow problems; see [2,1,3]. So far, however, the use of this methodology has been limited to discretizations using structured meshes in simple, two-dimensional geometries. The main challenge is the (approximate) solution of the velocity subproblem in the augmented Lagrangian formulation: while effective geometric multigrid methods have been developed for 2D structured meshes, the question remains open for the case of 2D and 3D unstructured meshes. One possible solution to this problem is to approximate the coefficient matrix of the velocity subproblem in the preconditioner with a more manageable one, in particular, one for which there exist efficient algebraic solvers. The problem is how to do this in such a way that the overall effectiveness of the preconditioner is preserved. In this talk I will present some variants of the augmented Lagrangian preconditioner that are applicable to rather general discretizations and geometries and retain excellent overall robustness and convergence properties.

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