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FAST CALCULATION OF ELASTIC FIELDS AROUD INCLUSIONS AND CRACKS IN COMPOSITE MATERIALS

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ABSTRACT

The problem of calculation of elastic fields around inclusions, cracks, and other defects in homogeneous elastic media is a crucial problem of micromechanics. If an inclusion has non-canonical form (ellipsoid and its limit forms) this problem may be solved only numerically. The finite element method (EFM) leads to the necessity of discretization of a large region that includes the inclusion, and the results of the calculations depend on the conditions on the boundary of this region. The problem may be reduced to the solution of volume integral equations over the region occupied only by the inclusion. For this method, the area of the solution turns out to be much smaller than by usage of the FEM.

The method of volume integral equations has several well known drawbacks. Firstly, the calculation of the elements of the matrix of the discretized problem involves numerical integration of highly singular functions (the kernels of the integral operator are Green's functions and its derivatives) that is an expensive procedure. Secondly, the matrix of the disretized problem is non-sparse, and storage of such matrices in the computer memory and operations with them are also very expensive. The method proposed in this work is free of these drawbacks.

The basic point of the method is the use of radial Gaussian approximating functions for the presentation of strain or stress fields inside an inclusion. The theory of approximation by Gaussian functions was developed in the works of V. Maz'ya et al. [1, 2]. The advantage of these functions is that action of many integral operators of mathematical physics on such functions may be obtained in a simple analytical form. As a result, the time of calculation of the elements of the matrix of the discretized problem is essentially reduced in comparison with the methods based on conventional approximating functions (e.g., polynomial splines). The method is mesh free in fact, and only coordinates of the nodes – centres of the Gaussian functions - are initial data for carrying out the method.

Essential saving in the computer time and memory is achieved if regular grids of nodes are used. In this case, the matrix of the discretized linear system proves to have the Teoplitz structure. As a result, the Fast Fourier Transform algorithms may be used for

the calculation of matrix-vector products in this case. This fact essentially reduces the time of the process of the iterative solution of the discretized problem when such a product is calculated at every step of the iteration process.

In this work, the solutions of 3D-problems of elasticity for an infinite homogeneous isotropic medium containing a plane crack of an arbitrary form, or an arbitrary heterogeneous inclusion are presented. The numerical results are compared with exact solutions for ellipsoidal inclusions and elliptical cracks. Some specific features of the method and details of its application are indicated and discussed. It is shown that the method allows us to obtain the numerical solution in a reasonable time for the grids with several millions nodes using modest modern workstations.

Application of this method to the solution of the electrostatic problems for media with inclusions may be found in [3, 4, 5].

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