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Numerical solutions of boundary value problems with distributional data in GFEM

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ABSTRACT

We investigate the numerical approximation of the solutions of the Poisson boundary value problem with Dirichlet boundary conditions when the data has low regularity and the Generalized Finite Element Method is used:

$$-\Delta u = f \text{ in } \Omega \quad u = g \text{ on } \partial\Omega,$$

for a smooth domain Ω. Our method is based on using approximate Dirichlet boundary conditions and polynomial approximations of the boundary. The boundary value problem is formulated in a weak sense in such a way as to allow data with low regularity (distributions). The resulting weak formulation satisfies the Babuška-Brezzi condition. The sequence of GFEM-spaces is required to satisfy a few natural assumptions. However, our approximating functions are not required to vanish on the boundary, and hence our GFEM spaces do not conform to one of the basic Finite Element Method (FEM) conditions. Nevertheless, we can use the usual definition of the discrete solutions. We obtain quasi-optimal rates of convergence for the sequence of GFEM approximations of the exact solution. We also extend our results to the inhomogeneous Dirichlet boundary value problem. Finally, we indicate an effective technique for constructing sequences of GFEM-spaces satisfying our assumptions by using polynomial approximations of the boundary.

See [1] for full details. See [2, 3, 4] for an introduction to GFEM and for related results.

REFERENCES

[1] I. Babuška, V. Nistor, and N. Tarfulea. “Approximate and low regularity Dirichlet boundary conditions in the generalized finite element method.” *M^3AS.*, Vol. **17**, 2115–2142, 2007.

[2] I. Babuška and V. Nistor. “Interior Numerical Approximation of Boundary Value Problems with a Distributional Data.” *Numerical Meth. PDEs*, Vol. **2**, 79–113, 2006.

[3] I. Babuška, U. Banerjee, and J. E. Osborn. “Survey of meshless and generalized finite element methods: A unified approach,” *Acta Numerica*, Vol. **2003**, 1–125, 2003.

[4] A. Duarte, D.-J. Kim, and D. M. Quaresma. “Arbitrarily smooth Generalized Finite Element approximations,” *Comput. Math. Appl. Mech. Engrg.*, Vol. **196**, 33–56, 2006.