## SOME STABILITY AND POST-BUCKLING BEHAVIOUR PROBLEMS OF THIN-WALLED STRUCTURES

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### ABSTRACT

A review of stability and post-buckling behaviour problems of thin-walled structures is presented. At first, the paradox of the critical load of torsional buckling of axially compressed I columns with variable flanges width is discussed. The classical theory of thin-walled beams with non-deformable cross-section is employed [1]. Using Pontryagin's maximum principle [2] it is proved that the critical loads of torsional buckling of column with variable flanges width may lay outside the limits described by the columns with constant maximum and minimum width of flanges. Optimal shapes of the flanges, within constant maximum and minimum width, for extreme values of the critical of torsional buckling are presented. Moreover, the post-buckling behaviour of the column is investigated and it is shown that all bifurcation points are symmetrical and stable.



# Fig.1. Simply supported I column subject to axial end loads and distributions of residual stresses $\sigma_0$

Comparison these results and the fully nonlinear analysis of shells confirms this paradoxical property of the torsional buckling load [3]. Secondly, some comparative study of effects of residual stresses (Fig.1) and torsional and warping stiffeners on the torsional buckling critical loads of thin-walled columns with bisymmetric cross-section are presented. The initial post-buckling equilibrium path is determined by utilizing a perturbation approach. Numerical examples dealing with an I column and a column with cruciform cross-section point out that a decrease of the critical load due to the residual stresses is possible. However, the bifurcation points established are symmetric and stable [4].

Next, the effect of initial curvature on the critical load of torsional buckling of I columns is studied by means of the classical theory of beams and the fully nonlinear theory of shells. The numerical examples given allow us to draw a conclusion that agreement between the results is very good and a reduction of the critical load due to these imperfections should be noticed.

Sensitivity analysis of the critical loads of torsional buckling of thin-walled I columns with respect to changes of the column flanges width, property of the column material and a change of the location of the column ends is carried out. The considerations are restricted to the single eigenvalues. Numerical examples given allow us to investigate an accuracy of the sensitivity analysis. Moreover, sensitivity analysis of the flexuraltorsional buckling load of I beam subjected to axial loads and bending moments with respect to the same design variables is presented. The first variation of the critical loads due to the design variable is derived. Numerical examples related to simply supported beams are` shown. It should be noticed that the variations can be also utilized in more complex cases, in which the stability analysis is performed with aid of the Finite Element Method.

Finally, the effects of nonlinearity of the column material and the restraints on the critical loads of flexural and torsional buckling and initial post-buckling behaviour are studied. The stability analysis is carried out by means of a perturbation approach. Some numerical examples dealing with I columns made of aluminium are shown. In order to apply the classical one-dimensional theory of beams to thin-walled frames and grids it is necessary to have a rule of distribution of bimoments in the structure nodes. Unfortunately, the bimoment is self-equilibrium force and it is impossible to obtain suitable the node equilibrium condition. One manner to overcome this difficulty is to use the superelement concept proposed in [6].

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