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Kinematically coupled algorithms for fluid-structure interaction in blood flow simulations

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ABSTRACT

Several techniques have been proposed for the numerical solution of fluid-structure interaction problems. Some of the most popuplar techniques are the Immersed Boundary Method, the Arbitrary Lagrangian Eulerian (ALE) approach, the Fictitious Domain method, the Lattice Boltzmann method, the Level Set Method and the Coupled Momentum Method.

To date, only *strongly coupled* (monolithic/implicit/semi-implicit) algorithms seem applicable to blood flow simulations [1, 2, 3, 4, 5, 6, 7]. These methods are definitely robust, but unfortunately they are generally quite expensive in terms of computational time. The multi-physics features of fluid-structure interaction strongly suggest to employ *partitioned algorithms* to split the coupled problem in simpler subproblems. Following the multi-physics, a common way to split the problem is to solve separately the fluid and the structure equations. This leads to *loosely coupled* (explicit) algorithms, widely used in aeroelasticity [8, 9, 10], that utilize only one fluid solver and one structure solver at every time step. It has been shown in [3] that when the densities of the fluid and of the structure are comparable, loosely coupled algorithms suffer from severe stability issues due to the "added mass effect". Unfortunately, this is the case in blood flow simulations.

In this talk, we will present novel numerical algorithms to solve the fluid-structure interaction in blood flow, which combine the stability properties of fully-coupled schemes with the low computational costs and implementation time of partitioned schemes. Instead of splitting the fluid from the structure, as in the classical partitioned schemes, we split the structure equation in its *hydrodynamical* and *elastic* parts. The hydrodynamical part consists of the fluid stress on the interface and will be treated together with the fluid equations. The remaining terms constitute the elastic part and will be treated separately.
The main ideas behind our splitting strategy are the following:

1. ***Cardinal role of the kinematic condition.*** We use the kinematic condition to rewrite the structure acceleration in terms of the fluid acceleration at the interface. This ensures a tight link between fluid and structure at each step and, in addition, it allows to rewrite the problem in a new form

involving only first-order differential operators in time. We then start from this first-order formulation to perform the time discretization by operator splitting. As a matter of fact, the theory of operator splitting is well developed only for first-order differential operators, while it has yet to be fully understood for higher order problems.

2. ***Proper treatment of non-dissipative sub-steps.*** It is our experience that it is crucial for the stability and accuracy of splitting schemes to treat properly non-dissipative sub-steps. The elastic part of the structure equation is essentially hyperbolic and therefore non-dissipative; it will be treated in a separate step, where we can use a non-dissipative solver.

Numerical results will be presented.

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