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Reduced basis and iterative algorithms for non-linear elastic thin shells.

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ABSTRACT

In this work, we propose an iterative linear solver for the linearized equations coming from the Newton-Raphson method. In structural mechanics, the computation of the non-linear solution, with a Newton-Raphson method for example, requires the solution of sparse linear systems of equations:

$$K^iU^i = F(U^j) \text{ with } i = 1, .., k \text{ and } j = 1, .., i-1 \quad (1)$$

with K^i designs a $N \times N$ symmetric matrix (the tangent matrix), U^i is the unknown displacement vector ($U^i \in \mathbb{R}^N$) and $F(U^j)$ is a load vector which depends on the previous solutions U^j with $j = 1, .., i-1$. In this work, problems (1) result from the discretization with the finite element method of the non-linear elastic thin shell equations. In moderate scale problems (less than 20 000 unknowns), problems (1) are solved by using direct triangulation, such as the Crout method for the considered problem. When the problem size increases, iterative methods are generally used. For the symmetric problem (1) the most useful method is the conjugate gradient method connected to preconditioning techniques. Whereas direct methods provide, after a knonwn number of operations, the exact solution of the initial problem (1), iterative methods generate a sequence of approximate vectors which converge to the desired solution. Generally, the choice between direct and iterative methods is made according to the size of the discretized problems. Nevertheless, within non-linear computations the choice of the solver is not as easy. Indeed, if the matrix K^i is constant during all the iterations i (within the Newton modified method or with an Asymptotic Numerical Method [1]), a decomposition of this matrix is carried out once and following problems need only backward and forward substitutions. So, the most computing time step is realized for the first problem and the following have low computing costs. With iterative methods, all the problems i need the same computing time and the total computational cost can be high. Nevertheless, as iterative methods require only product matrix-vector, the amount of stored data is generally less than with a direct method. In this work, we propose an iterative method to

Step	Number of iteration of the Newton's corrector	PM (n)	PCG	PCG IC[0]	PCG IC[1]
4	2	6 (15)	11	170	110
5	2	6 (17)	11	176	111
6	3	7 (21)	12	178	113
7	3	8 (25)	17	184	121
8	3	9 (29)	20	193	124
9	3	10 (33)	21	198	130
10	3	9 (37)	21	198	135

Table 1: Comparison of the average number of iterations to get the desired accuracy ($\eta = 10^{-4}$) on the linear problem (1) for the Proposed Method (PM) and the Preconditioned Conjugate Gradient method (PCG). (•) is the number of vectors of the reduced basis. IC[•] refers to the level of the incomplete Crout triangulation. 10 steps are necessary to get the non-linear response curves. The demanded accuracy on the non linear problem is 10^{-3}.

solve linear systems (1) where the matrix K^i is not identical for all the iterations i and have different right-hand side vectors $F(U^j)$. The key point of this method is to associate a direct and an iterative method. The direct method is used to solve a reduced size problem. The vectors used to build this reduced problem are issued from the previous solutions U^j with $(1, .., i-1)$. The second important point is to use within the iterative method a preconditioning matrix which is a matrix triangulated at a previous step. Usually, preconditioning techniques do not use full matrices but incomplete factorization. As complete triangulation requires consequent CPU time, the obtained preconditioning matrix is used for solving several linear problems (1), either for several Newton's iterations or for several steps of the prediction-correction scheme (Newton-Raphson method). Finally, as the convergence of the proposed iterative method can be slow (sometimes it can be divergent) a convergence accelerating technique is used[2].

To show the efficiency of the proposed iterative linear solver we applied it to the a classical geometrically nonlinear problem : a cylindrical shell with two diametrically opposed rectangular cut-outs. The number of unknowns for this example is equal to 5190. In table (1), we compare the average number of iterations to get the demanded accuracy on the linear problem (1) for 4 iterative methods : the proposed method (PM), the preconditioned conjugate gradient method with the same preconditionning matrix that the proposed method (PCG), the PCG with an incomplete Crout triangulation as preconditionning (PCG IC[*] with '*' means the number of terms computed for the incomplete factorization). One can see in table (1) that the use of the prediction matrix as a preconditioning technique is the correction phase is very efficient. Indeed when using a classical preconditioning method, such as IC[*], the number of iterations to get the demanded accuracy is roughly equal to 200 with IC[0] and 100 with IC[1]. Whereas with the prediction matrix as preconditioner, the PCG needs for all the linear problems solved less than 30 iterations. The results presented in table (1) show that the number of iterations is approximately twice as less with the proposed method than with the PCG (with the same preconditioning method).

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