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GENERALIZED EIGENVALUE PROBLEM FOR NONLINEAR STABILITY ANALYSIS

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ABSTRACT

Detection and calculation of critical point may be carried out using different methods [1-2]. First of all, one can distinguish “exact” methods based on “true” nonlinear equilibrium equations, from approximate methods based on some simplifications, i.e. artificial linearization of the problem. The first family of methods is especially useful as an additional tool for path following and continuation process. Precise estimation of critical load may be achieved in direct or indirect way [1]. The second family of methods enables prediction of critical load in advance (extrapolation), with or without evaluation of intermediate points on the equilibrium path. The most representative and commonly used method of this family is connected with so-called *initial buckling* eigenproblem, described as follows:

$$(\mathbf{K}_0 + \lambda \mathbf{K}_g)\,\mathbf{v} = 0\,. \tag{1}$$

$\mathbf{K}_0$ and $\mathbf{K}_g$ are matrices of initial and geometric stiffness respectively [3]. The solution to the above eigenproblem enables to establish both critical loads and corresponding buckling modes; however the accuracy of critical load predicted in this way may be very poor. The paper discusses successive improvements to this classical approach. The final conclusion of the investigation leads to the following generalized eigenvalue problem:

$$\left[\mathbf{K}(\eta_0) + \Delta\mathbf{K}(\eta_0)\frac{(\eta-\eta_0)}{\Delta\eta}\right]\mathbf{v} = \mathbf{0}\,, \tag{2}$$

in which the tangent stiffness matrix $\mathbf{K}$ is calculated in two successive points on the equilibrium path. Contrary to classical approaches $\mathbf{K}$ is parameterized here in different way (λ is not proper parameter in the vicinity of limit point hence the tangent stiffness matrix can not be treated as the function $\mathbf{K}(\lambda)$). Above parameterization may be based on “leading” displacement or the arc-length method.

Consider well known von Mises truss shown in Fig. 1. The nonlinear behaviour of this structure is described by the equilibrium path shown in Fig. 2. For simplicity, the calculations were carried out for $a = b = 1$ and $EA = 1$. Comparison of accuracy of predicted critical load using different methods is included in Tab. 1. The first column describes the applied method, starting parameter (λ_0 or η_0) and its increment. Two next columns include critical displacement (v_{cr}) and load (P_{cr}). The ratio and relative error of critical load is presented in two last columns. The table clearly shows the great improvement of accuracy using proposed approach. Contrary to classical methods the estimation is now understated (safer). More complex examples, including structures exhibiting buckling mode interaction [2], and more detailed discussion of the results will be presented in the full paper.

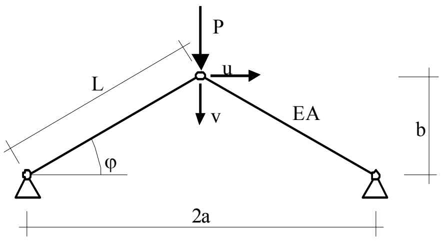

Fig. 1. Mises truss.

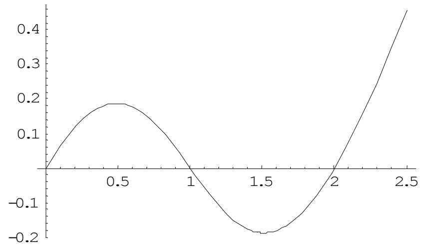

Fig. 2. Equilibrium path.

Conclusions: The paper has discussed different approaches for calculation of critical load. Successive improvements to the original eigenproblem describing initial buckling have been studied. The result of presented investigations is the new method for estimation of critical load without complete (and time consuming) determination of equilibrium path. The numerical examples have proved its high efficiency. It may successfully be applied both in bifurcation and load limit points.

Tab. 1. Comparison of accuracy of critical load for different methods.

Method	v_{cr}	P_{cr}	P_{cr}/P^*	*error*
Exact (v*, P*)	0.490175	0.187403	-	-
Initial BP	-	1.414210	7.55	654.6%
Linear BP	-	0.471405	2.52	151.5%
Quadratic BP	-	0.291344	1.55	55.5%
ULSP 0+0.1	-	0.363021	1.94	93.7%
ULSP 0.10+0.01	-	0.289520	1.54	54.5%
ULSP 0.18+.005	-	0.191697	1.02	2.3%
GSP 0+0.1	0.619114	0.173228	0.92	-7.6%
GSP 0.1+0.1	0.552731	0.184047	0.98	-1.8%
GSP 0.3+0.1	0.494929	0.187384	1.00	-0.01%
GSP 0.4+0.1	0.490080	0.187403	1.00	-4.1e-8

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