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Topological Derivative in Multi-scale Heat Conduction Models

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ABSTRACT

The macroscopic (or effective) heat conductivity of materials is a physical property of paramount importance in the design of mechanical, thermal and electronic components for a vast number of applications in civil, aerospace, biomedical, nuclear and electronics industries. In many circumstances, this property dictates the design approach and any improvements in component performance can only be achieved by means of suitable changes in the conductivity behaviour of the adopted materials. In this context, the ability to accurately predict the macroscopic conductivity from the corresponding microstructural properties becomes essential in the analysis and potential purpose-design and optimisation of the underlying heterogeneous medium. Methods for estimation of effective conductivity have been proposed and investigated, among others, by Germain *et al.* , 1983; Auriault, 1983; Auriault and Royer, 1993; Ostoja-Starzewski and Schulte, 1996; Wang *et al.* , 2006; and Jiang and Sousa, 2007. Of crucial importance to the potential optimisation of the conductive medium in this case, is the sensitivity of the effective conductivity to changes in the microstructure – and whose calculation, to the authors knowledge, has not been reported in the literature. This paper proposes a general analytical expression for the sensitivity of the two-dimensional macroscopic heat conductivity tensor to topological changes of the micro-structure of the underlying material. The macroscopic conductivity is estimated by means of a homogenisation-based multi-scale constitutive theory for steady-state heat conduction problems where, following closely the ideas of Germain *et al.* , 1983, the macroscopic temperature gradient and heat flux vectors at each point of the macroscopic continuum are defined as the volume averages of their microscopic counterparts over a Representative Volume Element (RVE) of material associated with that point. In this context, the estimated effective conductivity depends on the choice of constraints imposed upon the admissible temperature fields of the RVE, and the upper and lower bounds established by Ostoja-Starzewski and Schulte, 1996, can be obtained by suitable choices of constraints. The proposed sensitivity is a symmetric second order tensor field over the RVE that measures how the macroscopic conductivity estimated within the multi-scale framework changes when a small circular inclusion is introduced at the micro-scale. Its analytical formula is derived by making use of the concepts of *topological asymptotic expansion* and *topological derivative* (Sokolowski and Zochowski, 1999; Céa *et al.*

, 2000) within the adopted multi-scale theory. These relatively new mathematical concepts allow the closed form calculation of the sensitivity, whose value depends on the solution of a set of equations over the original domain, of a given shape functional with respect to an infinitesimal domain perturbation, like the insertion of holes, inclusions or source term. Among the methods for calculation of the topological derivative currenlty available in the literature, here we shall adopt the *topological shape-sensitivity method* proposed by Novotny *et al.* , 2003. In the present context, the variational setting in which the underlying multi-scale theory is cast is found to be particularly well-suited for the application of the topological derivative formalism. The final format of the proposed analytical formula is strikingly simple and can be potentially used in applications such as the sythesis and optimal design of microstructures to meet a specified macroscopic conductivity behaviour. In this work, initially we describes the multi-scale constitutive modelling approach adopted in the estimation of the macroscopic heat conductivity tensor which is based on the work by de Souza Neto & Feijóo, 2006, in the solids mechanics context. Next, we present the main result of the paper – the closed formula for the sensitivity of the macroscopic conductivity to topological microstructural perturbations. Here, a brief description of the topological derivative concept is initially given. This is followed by its application to the problem in question which leads to the identification of the required sensitivity tensor. A simple finite element-based numerical example is also provided for the numerical verification of the analytically derived topological derivative formula. The paper ends with some concluding remarks.

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