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Topological Derivative in Multi-scale Linear Elasticity Models

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ABSTRACT

Composite materials have become one of the most important classes of engineering materials. Then, the macroscopic mechanical behavior of this materials has a paramount importance in the design of mechanical components for a vast number of applications in civil, aerospace, biomedical and nuclear industries. In a broad sense, one can argue that much of material science is about topological and shape modifications on materials' microstructures. For example, changing the shape of the graphite inclusions in a cast iron matrix dramatically changes all the macroscopic properties of the material. In this context, the ability to accurately predict the macroscopic mechanical behavior from the corresponding microstructural properties becomes essential in the analysis and potential purpose-design and optimisation of the underlying heterogeneous medium. The goal of this design and optimization is to control the macroscopic material properties, either to improve them, or to build materials with prescribed properties. Of crucial importance to the potential optimisation of the medium in this case, is the sensitivity of the effective macroscopic parameters to changes in the microstructure. In this sense, we can cite many authors that have been building new microscopic topologies based on heuristics arguments, for example, microstructures with macroscopic negative Poisson's ratio (see, for instance, Almgreen, 1985 and Lakes, 1987). This paper proposes a general analytical expression for the sensitivity of the two-dimensional macroscopic elasticity tensor to topological changes of the microstructure of the underlying material. The macroscopic elasticity response is estimated by means of a homogenisation-based multi-scale constitutive theory for elasticity problems where, following closely the ideas of Germain *et al.* , 1983, the macroscopic strain and stress tensors at each point of the macroscopic continuum are defined as the volume averages of their microscopic counterparts over a Representative Volume Element (RVE) of material associated with that point. It is analogous to the multi-scale strategy presented, among others, by Miehe *et al.* , 1999 and Michel *et al.* , 1999 – and whose variational structure is described in detail in de Souza Neto & Feijóo, 2006. In this context and based on the axiomatic construction of multi-scale constitutive models, the proposed sensitivity leads to a symmetric fourth order tensor field over the RVE that measures how the macroscopic elasticity parameters estimated within the multi-scale framework changes when a small circular hole is introduced at the micro-scale. Its analytical formula

is derived by making use of the concepts of *topological asymptotic expansion* and *topological derivative* (Sokolowski and Zochowski, 1999; Céa *et al.* , 2000) within the adopted multi-scale theory. These relatively new mathematical concepts allow the closed form calculation of the sensitivity, whose value depends on the solution of a set of equations over the original domain, of a given shape functional with respect to an infinitesimal domain perturbation, like the insertion of holes, inclusions or source term. Among the methods for calculation of the topological derivative currenlty available in the literature, here we shall adopt the *topological shape-sensitivity method* proposed by Novotny et al. 2003. In the present context, the variational setting in which the underlying multi-scale theory is cast, is found to be particularly well-suited for the application of the topological derivative formalism. The final format of the proposed analytical formula is strikingly simple and can be potentially used in applications such as the synthesis and optimal design of microstructures to meet a specified macroscopic behavior. In this work, initially we describes the multi-scale constitutive framework adopted in the estimation of the macroscopic elasticity tensor. A clear variational foundation of the theory is established which is essential for the main developments to be presented later. Next, we presents the main result of the paper – the closed formula for the sensitivity of the macroscopic elasticity tensor to topological microstructural perturbations. Here, a brief description of the topological derivative concept is initially given. This is followed by its application to the problem in question which leads to the identification of the required sensitivity tensor. A simple finite element-based numerical example is also provided for the numerical verification of the analytically derived topological derivative formula. The paper ends with some concluding remarks.

REFERENCES

[1] R.F. Almgreen. "An Isotropic Three-Dimensional Structure with Poisson Ratio -1". *Journal of Elasticity*, Vol. **15**, 427-430, 1985.

[2] J. Céa, S. Garreau, Ph. Guillaume & M. Masmoudi. "The shape and Topological Optimizations Connection". *Computer Methods in Applied Mechanics and Engineering*, Vol. **188**, 713–726, 2000.

[3] E.A. de Souza Neto & R.A. Feijóo. "Variational foundations of multi-scale constitutive models of solid: Small and large strain kinematical formulation". *LNCC Research & Development Report*, Vol. **16/2006**, National Laboratory for Scientific Computing, Brazil, 2006.

[4] P. Germain, Q.S. Nguyen & P. Suquet. "Continuum thermodynamics". *Journal of Applied Mechanics, Transactions of the ASME*, Vol. **50**, 1010–1020, 1983.

[5] R. Lakes. "Foam Structures with Negative Poisson's Ratio".*Science, AAAS*, Vol. **235**, 1038–1040, 1987.

[6] J.C. Michel, H. Moulinec & P. Suquet. "Effective properties of composite materials with periodic microstructure: a computational approach". *Computer Methods in Applied Mechanics and Engineering*, Vol. **172**, 109–143, 1999.

[7] C. Miehe, J. Schotte, & J. Schröder. "Computational Micro-Macro Transitions and Overall Moduli in the Analysis of Polycrystals at Large Strains". *Computational Materials Science*, Vol. **16**, 372–382, 1999.

[8] A.A. Novotny, R.A. Feijóo, C. Padra & E. Taroco. "Topological Sensitivity Analysis". *Computer Methods in Applied Mechanics and Engineering*, Vol. **192**, 803-829, 2003.

[9] J. Sokolowski & A. Zochowski. "On the Topological Derivatives in Shape Optmization". *SIAM Journal on Control and Optimization*, Vol. **37**, 1251-1272, 1999.