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SCALABLE TOTAL BETI ALGORITHM FOR 3D CONTACT PROBLEMS

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ABSTRACT

The Finite Element Tearing and Interconnecting (FETI) domain decomposition method was originally proposed by Farhat and Roux [1] for parallel solving of the linear problems described by elliptic partial differential equations. Its key ingredient is decomposition of the spatial domain into non-overlapping subdomains that are ”glued” by Lagrange multipliers, so that, after eliminating the primal variables, the original problem is reduced to a small, relatively well conditioned, typically equality constrained quadratic programming problem that is solved iteratively. The time that is necessary for both the elimination and iterations can be reduced nearly proportionally to the number of the processors, so that the algorithm enjoys parallel scalability. Observing that the equality constraints may be used to define so called ”natural coarse grid”, Farhat, Mandel and Roux [2] modified the basic FETI algorithm so that they were able to adapt the results by Bramble, Pasciak and Schatz [3] to prove its numerical scalability, i.e. asymptotically linear complexity. The comprehensive review of the mathematical results related to the FETI methods may be found in the monograph by Tosseli and Widlund [4].

If the FETI procedure is applied to an elliptic variational inequality, the resulting quadratic programming problem has not only the equality constraints, but also the non-negativity constraints. Even though the latter is a considerable complication as compared with linear problems, it seems that the FETI procedure should be even more powerful for the solution of variational inequalities than for the linear problems. The reason is that FETI not only reduces the original problem to a smaller and better conditioned one, but it also replaces for free all the inequalities by the bound constraints. Promising experimental results by Dureisseix and Farhat [5] and Dostál et al. [6] supported this claim and even indicated numerical scalability of such methods. Recently, Dostál and Horák [7] used the FETI method with a natural coarse grid to develop a scalable algorithm for numerical solution of both coercive and semicoercive variational inequalities. The rate of convergence was given in terms of the effective condition number of the dual Schur complement of the stiffness matrix, which is known [2] to be bounded by CH/h, where C is a constant independent of the discretization and decomposition parameters h and H, respectively. The estimates did not assume any preconditioning except the projector to the natural coarse grid.

Since the the nonlinearity of the boundary variational inequality is limited to the boundary, it is natural to eliminate the interior unknowns by application of the boundary element method. We use the boundary element counterparts of the FETI and TFETI methods, the Boundary Element Tearing and Interconnecting (BETI) method introduced by Langer and Steinbach [8] and by its variant, the Total FETI (TFETI, also all floating) method introduced independently in thesis by Of and by Dostál et al. [9]. The core of our lecture is the report on our recent results on optimal algorithms for the solution of variational inequalities discretized by BETI and TBETI methods. We have thus extended our optimality results to the numerical solution of contact problems discretized by BETI and TBETI. The analytical engine of our theory is the spectral equivalence of the FETI and BETI dual Schur complements proved by Langer and Steinbach [8] and the optimality of our algorithms for the solution of bound and/or equality constrained quadratic programming problems [10, 11, 12]. The theory covers both the coercive and semicoercive problems, i.e., the problems with floating bodies. We conclude our lecture by numerical experiments that illustrate the efficiency of the methods presented in practice.

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