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STRESS ANALYSIS FOR THE DIRECT SHEAR OF ROCK MASSES ADOPTING A TRIANGULAR FINITE ELEMENT WITH AN EMBEDDED INTERFACE

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ABSTRACT

The mechanisms of the direct shear of materials are complicated. Therefore, it is sometimes necessary to estimate the macroscopic direct shear strength of materials as a practical problem. This paper suggests a method of finite element analysis for the estimation of the shear strength of rock masses, which is practically estimated through the use of typical in situ direct shear tests.

With the failure of rock masses, two kinds of failure should be considered. Firstly, the fractures of intact rock materials, which compose the macroscopic progressive failure, should be examined. Secondly, the sliding and the opening of the planes of weakness, which are distributed in rock masses from the beginning, should be considered. In this paper, the first kind of failure is regarded as the fundamental problem.

In the computation, the appearance of local cracks is firstly determined by comparing the stress values for the constant strain triangles (CST) and the failure criterion of the materials by Hoek [1]:

$$\sigma_1{}' = \sigma_3{}' + \sqrt{m\sigma_{\rm ci}\sigma_3{}' + s\sigma_{\rm ci}{}^2}$$

where $\sigma_1{}'$ and $\sigma_3{}'$ are the effective major and minor principal stress values, $\sigma_{\rm ci}$ is the compressive strength of the intact material, and s and m are the parameters representing the degree of damage to the macroscopic material.

As proposed by Bolzon[2], if local cracks appear, the applicable elements are to be replaced with triangular elements, each of which has an embedded interface, in order to represent the discontinuities of stress and the displacement across the cracks. The equilibrium of an elastic body Ω is stated in a weak form as

$$\int_{\Omega'} (\boldsymbol{C}\delta\boldsymbol{u})^{\rm T}\,\boldsymbol{\sigma}{\rm d}\Omega - \int_{\Omega'} \delta\boldsymbol{u}^{\rm T}\bar{\boldsymbol{b}}{\rm d}\Omega - \int_{\partial\Omega'_t} \delta\boldsymbol{u}^{\rm T}\bar{\boldsymbol{t}}{\rm d}S + \int_{\Gamma} \delta\boldsymbol{w}^{\rm T}\boldsymbol{t}{\rm d}\Gamma = 0$$

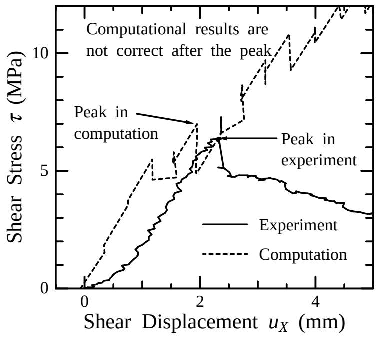

Figure 1: Relationships between shear displacement and shear stress

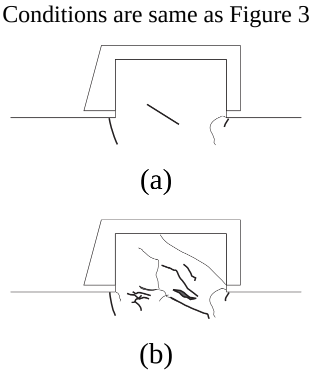

Figure 2: Crack propagations in the experiment: (a) Just after the peak and (b) In the residual state

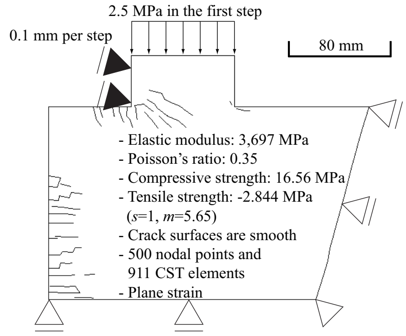

Figure 3: Crack propagations obtained from the computation: just after the peak

where an interface Γ, dividing Ω into the portions Ω^+ and Ω^-, is considered in the 2D domain. Γ^+ and Γ^- are the sides of Γ relevant to Ω^+ and Ω^- respectively. Body force $\bar{\boldsymbol{b}}$ acts in Ω, while loads $\bar{\boldsymbol{t}}$ and displacement $\bar{\boldsymbol{u}}$ are respectively assigned to the boundaries $\partial\Omega_t$ and $\partial\Omega_u$, respectively. The tractions applied to the surfaces Γ^+ and Γ^- are respectively denoted by $\boldsymbol{t}^+$ and $\boldsymbol{t}^-$, respectively, and the equilibrium requires that $\boldsymbol{t}^- = -\boldsymbol{t}^+ \equiv \boldsymbol{t}$. Displacement field $\boldsymbol{u}$, inside Ω, is conceived as the sum of the regular part $\tilde{\boldsymbol{u}}$ and the possibly discontinuous part $\hat{\boldsymbol{w}}$. $\hat{\boldsymbol{w}}$ is described separately as $\boldsymbol{w}^\pm$ in $\Omega^\pm$, and the difference $\boldsymbol{w} \equiv \boldsymbol{w}^+ - \boldsymbol{w}^-$. Vector $\boldsymbol{\sigma}$ includes the components of the Cauchy stress tensor, and $\boldsymbol{C}$ represents the differential operator of linear compatibility. $\delta\boldsymbol{u}$ and $\delta\boldsymbol{w}$ represent the virtual displacement, and $\Omega' = \Omega - \Gamma$. The discretized version of of the above equation is

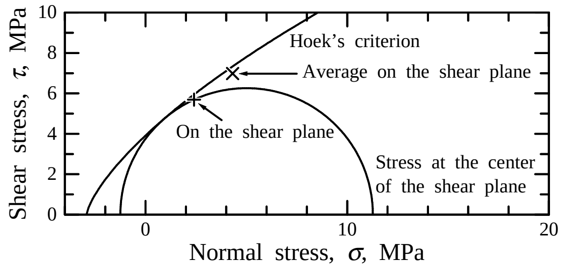

Figure 4: Stress diagram at the peak

$$\begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{uw}^+ & \boldsymbol{K}_{uw}^- \\ \boldsymbol{K}_{wu}^+ & \boldsymbol{K}_{ww}^+ & \boldsymbol{0} \\ \boldsymbol{K}_{wu}^- & \boldsymbol{0} & \boldsymbol{K}_{ww}^- \end{bmatrix} \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{W}^+ \\ \boldsymbol{W}^- \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{P}_u \\ \boldsymbol{P}_w^+ \\ \boldsymbol{P}_w^- \end{Bmatrix} - \begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{T} \\ -\boldsymbol{T} \end{Bmatrix}$$

where $\boldsymbol{U}$ and $\boldsymbol{W}^\pm$ are the discritized versions of $\tilde{\boldsymbol{u}}$ and $\boldsymbol{w}^\pm$, and $\boldsymbol{P}_u$, $\boldsymbol{P}_w^\pm$ and $\boldsymbol{T}$ are those of $\bar{\boldsymbol{b}}$ and $\bar{\boldsymbol{t}}$ for Ω', $\bar{\boldsymbol{b}}$ and $\bar{\boldsymbol{t}}$ for $\Omega^\pm$, and $\boldsymbol{t}$ on Γ. All the $\boldsymbol{K}$ represent the stiffness constructed by dividing the original CST, Ω, into a CST, Ω^-, and a 4-node isoparametric quadrilateral, Ω^+. In this examination, Γ ran through the major principal axis at the failure of each element.

With this procedure, the model tests which simulate practical in-situ rock shear tests are analyzed. The results obtained from the plaster model tests and the computations are shown in Figures 1 through 3. From these results, the stress paths during the tests are obtained, and the mechanisms of the appearance of the macroscopic strength values are interpreted with reference to the failure criterion of the materials, as shown in Figure 4.

REFERENCES

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