

## APPLICATION OF DISCONTINUOUS GALERKIN METHOD TO INTERFACE FRACTURE PHENOMENA

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**Key Words:** *Discontinuous Galerkin method, Interface fracture, Cohesive cracks.*

### ABSTRACT

The mechanical behavior of high-performance materials such as composite materials is greatly influenced by the interface fracture phenomena which usually originate from geometric discontinuities and material defects. A standard way to solve interface fracture problems within the finite element methods consists of inserting the interface cohesive elements between the standard volume elements at places where cracking is expected to occur [1].

However, the insertion of cohesive elements introduces a fictitious compliance in the structure. The magnitude of the artificial compliance is primarily related to the initial slope of the traction-separation law: a stiffer slope introduces a higher initial rigidity between neighbouring bulk elements, thus resulting in a smaller fictitious compliance. On the other hand, a high elastic stiffness of the cohesive surface compared to the elastic stiffness of the bulk material can result in artificial oscillations prior to the opening of the cohesive surfaces.

As an alternative to cohesive interface elements approach, in this paper, a class of nonsymmetric/ symmetric discontinuous Galerkin methods with interior penalties for interfacial fracture problems is presented. The discontinuous Galerkin methods are widely used in fluid mechanics, but recently, a significant number of results have been obtained in the area of solid mechanics [2, 3, 4, 5, 6]. The class of methods has a number of advantages, including flexibility and stability with respect to adaptivity of mesh and polynomial approximation.

The discontinuous Galerkin formulation presented in this paper allows for discontinuities in the displacement field across the interfaces. Within this formulation, the Dirichlet boundary conditions are imposed weakly instead of being built into the finite element space [3]. In order to implement the cohesive model within the discontinuous Galerkin formulation, the tractions on the element boundaries are computed using irreversible cohesive constitutive laws [6]. The cohesive constitutive law relates the interface (boundary) traction components to the displacement jumps components. When the normal and tangential components of the traction vector acting on the element boundaries exceed the maximum cohesive strength of the material, a discontinuity is introduced.

Based on the nonsymmetric/symmetric discontinuous Galerkin formulation a finite element code in C++ was developed. Numerical examples are presented to demonstrate the accuracy, stability and robustness of the proposed implementations concerning the interface cohesive crack propagation. Emphasis is placed on the application of different numerical integration schemes and their influence on the element performance. Moreover, the influence of the stabilization parameters on the numerical results is investigated.

The computational results show that the discontinuous Galerkin method can handle cohesive cracks very naturally with some advantages over the other methods, including good stability and consistency, and absence of traction oscillations and spurious reflections.

### ACKNOWLEDGEMENTS

The author would like to thank the ANCS for the support through the grant CEEEX 5850/2006.

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