

## MICROMECHANICS OF PARTICULATE SOLIDS AND ITS APPLICATION TO FAULTING PROCESS

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### ABSTRACT

Geo-materials such as soils and rocks can be idealised as an aggregate of identical particles. Their mechanical properties are of great interest in many industrial applications. In particular, the shearing of an assembly of particles between two plates is an idealized analogue to faulting processes in the Earth's crust.

A geometrical description for granular medium is shown in Fig.1. It is assumed that the micro-structural deformation in granular materials is the effect of sliding contact. The movement of a single particle in the assembly can cause changes in void fraction and contact force distribution. In addition, the asymmetric contact force distribution around a particle can induce micro-rotation.

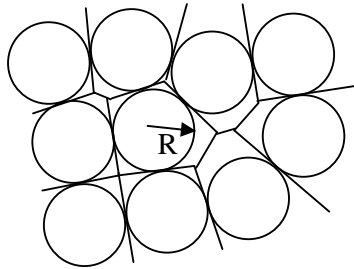


Figure 1 Granular packing

For a particle sited inside an aggregate, its macroscopic behaviour exhibits the features of Cosserat's continuum. Based on the energy-based definition of effective properties, we have

$$\sum_k F_k \Delta u_k = \frac{1}{2} \int_V (\sigma_{ij} \varepsilon_{ij} + \sigma_{[ij]} \gamma_{[ij]} + \mu_i \kappa_i) dV \quad (1)$$

where  $F_k$  is contact stresses and  $\Delta u_k$  is relative displacement;  $\varepsilon_{ij}$  is the symmetric strain conjugate to the symmetric stress,  $\sigma_{ij}$ ;  $\gamma_{[ij]} = e_{ijk} \omega_k$  is the anti-symmetric part of strain conjugate to the anti-symmetric stress,  $\sigma_{[ij]}$ , and  $\kappa_i$  is the curvature conjugate to the couple stresses  $\mu_i$ .

In addition, the contact force distribution is assumed to be symmetric to a director vector  $\mathbf{m}$ , which presents the direction of force chain at the particle level. Furthermore, the tangential contact stress is linear to the tangential displacement. We can found the work done by the contact force for a single particle as follows [1]:

$$\begin{aligned} \langle \bar{w} \rangle = & KR^2 \left[ \frac{4}{15} \varepsilon_{\alpha\beta} \varepsilon_{\alpha\eta} m_\beta m_\eta + \frac{4}{5} \varepsilon_{\alpha\beta} \varepsilon_{\alpha\beta} - \frac{4}{15} \varepsilon_{\alpha\alpha} \varepsilon_{\alpha\alpha} + \frac{8}{3} e_{\alpha\beta 3} \psi_3 \varepsilon_{\alpha\eta} m_\beta m_\eta \right. \\ & \left. + 4\psi_3 \psi_3 + \frac{4R^2}{3} \psi_{3,\alpha} \psi_{3,\beta} (m_\alpha m_\beta + \delta_{\alpha\beta}) \right] + \frac{2PR^2}{3} (\varepsilon_{\alpha\alpha} + \varepsilon_{(\alpha\beta)} m_\alpha m_\beta) \end{aligned} \quad (2)$$

where  $P$  is the hydrostatic stress related to pore pressure,  $K$  is the slope between tangential force and displacement;  $\psi_3$  is the particle rotation.

Based on thermodynamics theory, we can construct the averaged stress-strain relations and two evolution equations for the director vector and the void fraction. The details for their expressions can be found in the previous study [1].

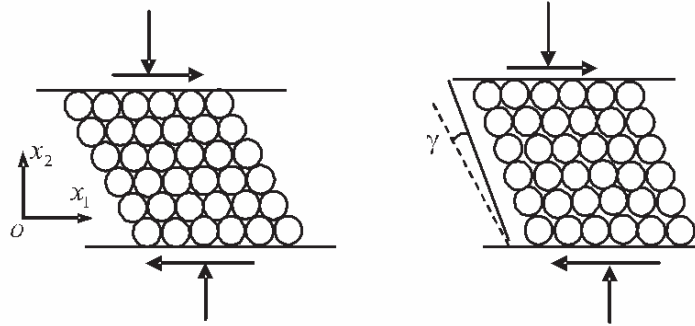


Figure 2: An ideal densely packed assembly of identical circular particles under simple shear: (a) Left: in its initial state and (b) Right: in a dilated state

Consider the shearing of an ideal granular packing in Fig. 2. For any small angular increment  $d\gamma$ , the director can be determined by the shear strain as follows

$$dm_1 = -2m_2 d\varepsilon_{12} \quad (3)$$

In addition, the void fraction is controlled by

$$df = \frac{\pi m_1}{2m_2^2} d\varepsilon_{12} \quad (4)$$

If the director angle varies from  $-\pi/6$  to  $\pi/6$ , the dilatation in the vertical direction and the linear shear stress and strain relation are provided in Figs. 3 and 4, respectively.

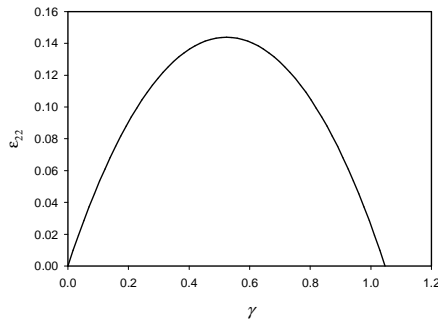


Figure 3: Vertical strain

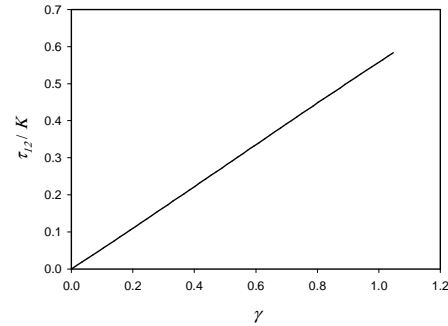


Figure 4: Shear stress

## REFERENCES

- [1] Zhang, X., Jeffrey, R. G. and Mai, Y-W., 2006. A micromechanics-based Cosserat-type model for dense particulate solids. *Zeitschrift für angewandte Mathematik und Physik (ZAMP)* 57, 682-707.