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HEAT WAVE SIMULATION IN MICRO-BARS WITH THE FINITE ELEMENT METHOD

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ABSTRACT

In this study we perform a finite element solution of the 1D ‘telegraph’ equation, which describes the formation of heat waves in a solid bar. Such waves are not predicted by the classical parabolic heat equation which implies that heat is dissipated with infinite speed. In order to overcome this paradox, several investigators have proposed a modified Fourier law, known as the Maxwell-Vernotte-Cattaneo law. The use of this modified law returns a hyperbolic equation. In our analysis we have considered a rectangular bar of small thickness with adiabatic ends. A laser pulse of small duration acts on a certain point of the bar, producing a localized heat flux q_0. This thermal loading generates thermal waves, traveling through the bar with finite speed. Thermal losses due to convection from the free surface are taken into account, so actually the bar acts like a thermal fin eqn. (1).

$$k\frac{\partial^2\theta}{\partial x^2}-\frac{2h}{b}\theta=\left(\rho C_P+\frac{2h\tau}{b}\right)\frac{\partial\theta}{\partial t}+\tau\rho C_P\frac{\partial^2\theta}{\partial t^2}-\frac{1}{b}\left(q_o+\tau\frac{\partial q_o}{\partial t}\right) \tag{1}$$

The analysis of the phenomenon has been conducted with the use of both linear and quadratic finite elements. The laser pulse has been simulated as the difference of two Heaviside step functions $H(t)-H(t-t_P)$, where t_P is the (variable) duration of the pulse. For the inertial term consistent matrices have been employed. The discretized equation is:

$$\mathbf{M}\overset{\circ\circ}{\boldsymbol{\theta}}+\mathbf{C}\overset{\circ}{\boldsymbol{\theta}}+\mathbf{K}\boldsymbol{\theta}=\mathbf{f} \tag{2}$$

Time integration has been performed with the use of the Newmark method.

Solutions of the thermal field along the bar for various time instants have been

obtained. Such solutions have several applications in micro-mechanics and micro-machining, where the domains are of very small characteristic lengths.

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