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Energy-Based Analysis of Mutual Entrainment in Vibro-Exciters on Oscillatory Base

*** Yuuichi YOKOI[1], Yoshihiko SUSUKI[2] and Takashi HIKIHARA[3]**

[1,2,3] Department of Electrical Engineering, Kyoto University
[3] Photonics and Electronics Science and Engineering Center, Kyoto University
Katsura, Nishikyo, Kyoto, 615–8510, JAPAN
{[1] yokoi, [2] susuki}@dove.kuee.kyoto-u.ac.jp, [3] hikihara@kuee.kyoto-u.ac.jp

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ABSTRACT

Synchronization has been extensively investigated for interacting dynamical systems [1,2]. In the systems, self-sustained oscillators synchronize in a common frequency by exchanging their stored energy through coupling. This energy exchange is crucial for understanding the phenomena. However, the energy aspects have hardly been used for analysis of the characteristics except in a few references [3,4]. The purpose of this paper is to reveal the relationship between phase and energy exchange in an entrained mechanical system. That is, an analysis of mutual entrainment is discussed in vibro-exciters on oscillatory base through energy exchange.

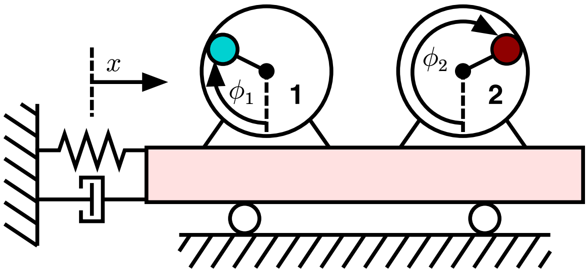

Figure 1: Vibro-exciters on oscillatory base [4].

Figure 1 shows the schematic configuration of two identical vibro-exciters on an oscillatory rigid base. This model is proposed by Blekhman [4]. The vibro-exciters are driven by external torque and rotate in clockwise direction. The rigid base is constrained to move in one dimension and is connected with an immovable foundation through elastic and damping elements. In the figure, $\phi_i\,(i = 1, 2)$ represents the angular displacement of vibro-exciter i, and x stands for the linear displacement of rigid base. A dimensionless form for the governing equations of motion is given by

$$\begin{cases} \dot{\phi}_i = \omega_i, \quad \dot{\omega}_i = -k\omega_i - \sin\phi_i + T_i + \varepsilon \dot{y}\cos\phi_i, \\ \dot{x} = y, \qquad \dot{y} = -\beta y - \gamma x + \displaystyle\sum_{i=1}^{2} \left(\dot{\omega}_i \cos\phi_i - \omega_i^2 \sin\phi_i\right), \end{cases} \tag{1}$$

where k denotes the damping coefficient for vibro-exciters, β the damping coefficient for rigid base, γ the elastic coefficient for rigid base, and T_i the external torque rotating vibro-exciter i. The overdot denotes the time differentiation. The parameter ε varies the coupling strength between vibro-exciters and rigid base. The parameters are set at $k = 0.3$, $\beta = 0.5$, $\gamma = 1.0$, and $T_1 = 1.5$.

Figure 2 shows the energy flow in the system. In the figure, the system can be decomposed into several components. In particular, the components represent three subsystems symbolized by Σ, external sources E, and dissipation elements D. Σ_1 (or Σ_2) corresponds to the vibro-exciter 1 (or 2) and Σ_X the rigid base. W denotes the exchanged energy among Σ, E, and D. Under harmonic entrained states, the following equality holds as an energy balance relation with Σ_i $(i = 1, 2)$:

$$0 = W_{\mathrm{E}\to i} - W_{i\to \mathrm{D}} - W_{i\to X}. \qquad (2)$$

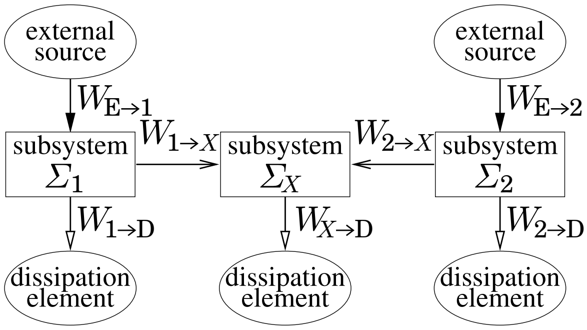

Figure 2: Energy flow in system.

In the right-hand side, the first term $W_{\mathrm{E}\to i}$ stands for the energy supplied to Σ_i by external torque. The second term $W_{i\to \mathrm{E}}$ represents the dissipated energy with damping coefficient. The last term $W_{i\to X}$ corresponds to the energy exchanged between Σ_i and Σ_X. When $W_{X\to i} > 0$, Σ_X does positive work to Σ_i. For Σ_X, an equality similar to (2) is also derived.

Figure 3(a) shows response curves for phase difference θ_{21} $(\triangleq \phi_2 - \phi_1)$, and (b) shows relationship between θ_{21} and exchanged energy $W_{2\to 1}$ $(\triangleq -W_{X\to 1})$. In Fig. 3, solid lines denote the response curves for stable solutions, and broken lines for unstable solutions. Figure 3(a) shows that the two

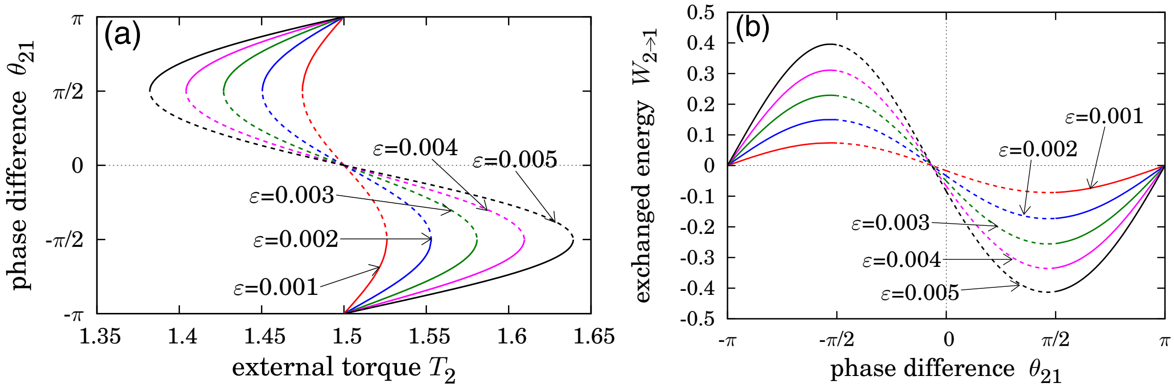

Figure 3: (a) Response curves for phase difference θ_{21}, (b) relationship between phase difference θ_{21} and exchanged energy $W_{2\to 1}$.

vibro-exciters synchronize in an anti-phase, and that the phase difference is a function of the external torque T_2. In Fig. 3(b), the energy has positive value when the phase difference becomes negative for stable solutions. This feature implies that if vibro-exciter 1 leads 2 with respect to phase, Σ_2 supplies energy to Σ_1 under harmonic entrained states. In addition, the energy exchange between vibro-exciters is a function of their phase difference. Consideration of the energy exchange possibly allows us to derive a phase equation for the mutual entrainment. The detail will be discussed in the final presentation.

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