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ADVANCES IN SOLUTION OF CLASSICAL GENERALIZED EIGENVALUE PROBLEM

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ABSTRACT

After the modelling by the Finite Element Analysis (FEA), some of the lowest eigenpairs of the following generalized algebraic eigenvalue problem are often required for a structure dynamic problem:

$$\mathbf{K}\varphi - \lambda \mathbf{M}\varphi = 0 \tag{1}$$

Owing to the growing size of the eigenvalue problem and the growing number of eigenvalues desired, solution methods of iterative nature are becoming more popular than ever, which however suffer from low efficiency and lack of proper convergence criteria. Although most commercial FEA packages offer different algorithms for the solution of eigenvalue problems, the results in general are not satisfactory due to the following problems: missing of eigenvalues; generation of false eigenvalues; generation of incorrect free modes and sometimes convergence has achieved for the eigenvalues, but not for the eigenvectors.

Traditionally, the key technique to accelerate the convergence for solution of Eq. (1) by iterative algorithm is using the shifting. The shifting cost depends on the solution efficiency for the following equation:

$$(\mathbf{K} - \mu \mathbf{M})x = \mathbf{LDL}^T x \tag{2}$$

First of all, a well developed cell storage sparse solver[2] is incorporated in this paper, instead of using the conventional skyline scheme. Very high efficiency is achieved using such an approach. Averagely the speed is one order up. This arising of the efficiency is such great that the shifting technology is realized and the Sturm Sequence Check for avoiding the losing of eigenvalues is always possible.

Secondly, after the discussion of the convergence criteria the following mode-error criterion is used:

$$\frac{|\mathbf{K}\tilde{\varphi}_k - \tilde{\lambda}_k \mathbf{M}\tilde{\varphi}_k|}{|\mathbf{K}\tilde{\varphi}_k|} \approx \frac{|\mathbf{K}\tilde{\varphi}_k - \tilde{\lambda}_k \mathbf{M}\tilde{\varphi}_k|}{|\tilde{\lambda}_k \mathbf{M}\tilde{\varphi}_k|} \le \varepsilon_\varphi \tag{3}$$

The criterion Eq. (3) includes not only the information of the eigenvalues but the eigenvectors. So it will be more stable to avoid generation of incorrect free modes and

also promote the convergence of the eigenvalues and eigenvectors reached simultanuousely. Many numerical examples showed that using Eq. (3) as the convergence criterion, more stable convergence characteristics can be achieved. On the other hand, the axisymmetric structure example with 6 rigid modes, multiple and closed eigenvalues shows that the Lanczos method in ANSYS, which used the criterion of relative eigenvalue error, failed to recognize 6 rigid body modes correctly. The subspace iteration in ANSYS failed to find all the multiple eigenvalues. Four multiple eigenvalues were lost in calculation of the 10 eigenvalues, resulting in the failure of iterations. Also, no convergence was achieved in calculation of the 20 eigenvalues.

Thirdly, the Subspace Iteration Method with an aggressive shifting strategy as shown in Fig. 1 is proposed. This aggressive shifting is a deeper shifting than existing shifting strategy like μ_B (Bathe), μ_W (Wilson) and μ_G (Gong). It can be used in any of iterative algorithms to accelerate the convergence rate.

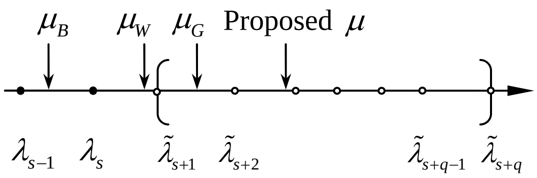

Fig. 1 An aggressive shifting strategy

After comparison of subspace iteration, iterative Ritz vector and iterative Lanczos methods, the iterative Ritz vector method appears to be the most efficient one, and the subspace iteration the slowest.

Using the “mode error” convergence criterion, instead of the eigenvalue-only criterion, a more stable and efficient iteration strategy for all three iterative methods was established. The possibility of losing the eigenvalues and creating the ghost eigenvalues and the bad free rigid body mode is much lower.

An aggressive shifting strategy is presented for the subspace iteration method. A posterior Sturm’s sequence check is necessary. If the aggressive shift is in trouble, then a backward shifting can be adopted to solve the problem. This technique of course can be used for any of iterative method with shifting strategy.

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