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TOTAL FETI METHOD FOR PARALLEL SOLUTION OF CONTACT SHAPE OPTIMIZATION PROBLEMS

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ABSTRACT

The contact shape optimization problems is one of the computationally most challenging problems. The reason is that not only the cost function is a nonlinear implicit function of the design variables, but that its evaluation requires also a solution of the highly nonlinear variational inequality which describes the equilibrium of a system of elastic bodies in mutual contact. Since the cost function must be evaluated many times in the solution process, it is obvious that the solution of contact problem is a key ingredient of any effective algorithm for the solution of contact shape optimization problems.

The approach that we propose here is based on the Finite Element Tearing and Interconnecting (FETI) domain decomposition method, which was originally proposed by Farhat and Roux [1] for parallel solving of the linear problems described by elliptic partial differential equations. Its key ingredient is decomposition of the spatial domain into non-overlapping subdomains that are "glued" by Lagrange multipliers, so that, after eliminating the primal variables, the original problem is reduced to a small, relatively well conditioned, typically equality constrained quadratic programming problem that is solved iteratively. The time that is necessary for both the elimination and iterations can be reduced nearly proportionally to the number of the processors, so that the algorithm enjoys parallel scalability. Observing that the equality constraints may be used to define so called "natural coarse grid", Farhat, Mandel and Roux [2] modified the basic FETI algorithm so that they were able to prove its numerical scalability, i.e. asymptotically linear complexity.

If the FETI procedure is applied to an elliptic variational inequality, the resulting quadratic programming problem has not only the equality constraints, but also the non-negativity constraints. Even though the latter is a considerable complication as compared with linear problems, it seems that the FETI procedure should be even more powerful for the solution of variational inequalities than for the linear problems. The reason is that FETI not only reduces the original problem to a smaller and better conditioned one, but it also replaces for free all the inequalities by the bound constraints [3]. Recently, Dostál and Horák [4] used the FETI method with a natural coarse grid to develop a scalable algorithm for numerical solution of both coercive and semicoercive variational inequalities.

In this talk, we exploit the parallel implementation of our scalable algorithm for contact problem to the minimization of the the compliance of the system elastic bodies subject the volume constraint and some additional constraints [5, 6, 7]. We start our exposition by recalling some theoretical results and formulae for derivatives of the solution with respect to the design variables. In particular, it turns out that the derivatives of the solution may be evaluated by the solution of variational inequalities with the same operator as the state problem. After identifying the subdomains with the bodies of the system and discretization, we describe our Total FETI (TFETI, also all floating) method introduced independently in thesis by Of and by Dostál et al. [8]. TFETI based domain decomposition algorithm for the solution of the resulting variational inequalities in two steps. First, using the duality theory, the problem to find the minimum of the energy functional subject to the kinematically admissible displacements is reduced to the contact interface. Then we exploit an efficient algorithm for the solution of the quadratic programming problems with simple bounds and possibly some equalities. An especially attractive feature of this approach is not only high precision of the gradient, but also the fact that relatively expensive decomposition of the stiffness matrices of the subdomains is carried out only once for each update of the design variables. Moreover, the decomposition update concerns only the subdomains affected by the update and we usually have good initial approximations for the solution. The efficiency of the proposed algorithms will be demonstrated on the compliance minimization of the Hertz problem.

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