Venice, Italy

ABSTRACT TITLE

Multiobjective optimization approach for model parameter extraction in gliding motility assays

* Florin Fulga¹, Cristinca Fulga² and Dan V. Nicolau³

¹ Department of Electrical Engineering and Electronics, University of Liverpool Liverpool L69 3GJ, UK fulga@liverpool.ac.uk, www.liv.ac.uk ² Dept. of Mathematics, Acad. of Economic Studies Piata Romana 6, 010174, Bucharest, Romania fulga@csie.ase.ro, www.ase.ro ³ Department of Electrical Engineering and Electronics, University of Liverpool Liverpool L69 3GJ, UK dnicolau@liverpool.ac.uk, www.liv.ac.uk

Key Words: Motility Assays, Multiobjective Optimization.

ABSTRACT

Over the past decade, rapid progress in optimization-based estimation and control have led to new approaches in the study of gliding assays for motor proteins. Motor proteins are enzymes that convert chemical energy into mechanical work having the ability to generate force and to undergo directed motion.

This talk focuses on the model of interaction molecular track – protein motor. The model includes a number of parameters depending on the basic assumptions. This paper describes a method of determining these parameters using the information from the gliding motility assays. The shape of the molecular track is captured at definite time intervals and discretized. The stochastic differential equation describing the time evolution of the molecular track is numerically solved and the solution sampled at the same time intervals.

Let $\overrightarrow{r}_{t,f}^{m}(s) = x_{t,f}^{m}(s) \overrightarrow{i} + y_{t,f}^{m}(s) \overrightarrow{j}$, $s \in [0,1]$ be the parametrization of the measured curve describing the shape of the molecular track $f \in \{1, 2, ..., F\}$ at the time $t \in \{0, 1, ..., T\}$; let $\overrightarrow{r}_{t,f}^{c}(s, p_{f}, q) = x_{t,f}^{c}(s, p_{f}, q) \overrightarrow{i} + y_{t,f}^{c}(s, p_{f}, q) \overrightarrow{j}$, $s \in [0,1]$ be the calculated parametrization obtained from the solution of the stochastic differential equation of the movement of the molecular track $f \in \{1, 2, ..., F\}$ at $t \in [0, T]$, where $p_{f} = (p_{1,f}, ..., p_{k_{f},f}) \in \mathbb{R}^{k_{f}}$ is the parameter-vector of the molecular track f and $q = (q_{1}, ..., q_{m}) \in \mathbb{R}^{m}$ is the parameter-vector common to all molecular tracks. In order to use the methods of nonlinear optimization, an objective function based on the difference between the measured shape of the molecular track and the calculated shape for different times is constructed. For every molecular track $f \in \{1, 2, ..., F\}$ and each time $t \in \{0, 1, ..., T\}$ the absolute value of the difference between the measured and the calculated parametrization is considered

$$\int_{0}^{1} \left(\left| \overrightarrow{x}_{t,f}^{m}\left(s\right) - \overrightarrow{x}_{t,f}^{c}\left(s,p,q\right) \right| + \left| \overrightarrow{y}_{t,f}^{m}\left(s\right) - \overrightarrow{y}_{t,f}^{c}\left(s,p,q\right) \right| \right) ds$$

where $p = (p_1, ..., p_F) \in R^K$ and $K = \sum_{f=1}^F k_f$. This difference could be interpreted as the distance between the measured and the calculated parametrization and is denoted by $d_{t,f}(p_f, q)$; the sum of the distances for each molecular track during the time interval [0,T] is $d_f(p_f,q) = \sum_{t=0}^T d_{t,f}(p_f,q)$. The goal is to find a vector of parameters $(p,q) \in R^K \times R^m$ such that the calculated parametrization had a minimal deviation from the measured parametrization. Therefore, the multiobjective optimization problem to be solved is:

$$\begin{cases} \min_{\substack{(p,q)\in R^{K}\times R^{m}}} \left(d_{1}\left(p_{1},q\right),d_{2}\left(p_{2},q\right),...,d_{F}\left(p_{F},q\right)\right)\\ \underline{p_{f}} \leq p_{f} \leq \overline{p_{f}}, \ f = \overline{1,F}\\ \underline{q} \leq q \leq \overline{q}. \end{cases}$$

where $\underline{p_f}$, $\overline{p_f}$ and \underline{q} , \overline{q} are the lower, respectively the upper limits for the parameter vector p_f , respectively q.

Methodological approaches in multiobjective optimization are mostly based on the calculation of one or several, usually efficient solutions which can be interpreted as compromise solutions. Only for specific types of the general multiobjective optimization problem, e.g, linear multiobjective optimization problems, have algorithms been developed for computing the complete efficient set. In this paper an efficient method for the multiobjective optimization problem is proposed. The problem has some specific features: the objective functions depend on local variables p_f , $f = \overline{1, F}$ and global variables q. The algorithm proposed in this paper coordinates the solution to the F subproblems to find the minimizer to the original problem. The coordination is carried out by the master problem, an optimization problem whose objective function is defined using information gathered from the subproblem solutions. If the vector global variables is set to a fixed value, the problem breaks into F independent subproblems. At each iteration of the optimization algorithm solving the master problem, all of the F subproblems are solved and information is exchanged between the master problem and the subproblems. The master problem is used to find the optimal value of the global variables.

REFERENCES

- [1] T. Duke, T. E. Holy and S. Leibler. "*Gliding assays*" for motor proteins: a theoretical analysis, Phys. Rev. Lett., Vol. **74(2)**, 330-334, 1995.
- [2] C. Fulga. "Decentralized cooperative optimization for multi-criteria decision making". *LNCIS, Adv. in Cooper. Ctrl. & Optimization*, Eds. M. J. Hirsch et al., Vol. 369, 65–80, Springer Berlin/Heidelberg 2007.
 - [3] K. Miettinen. *Nonlinear multiobjective optimization*, Kluwer Academic Publishers, Boston, 1999.