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SEMIANALYTICAL VERSUS NUMERICAL SENSITIVITY ANALYSIS: A CASE STUDY FOR THE CALIBRATION OF MATERIAL MODELS BASED ON MIXED FORMULATIONS AT LARGE STRAINS

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ABSTRACT

Material models contain characteristic parameters or functions which, in general, are not measurable directly but can only be identified analyzing their *effects* on the evolution of mechanical quantities. Consequently, the calibration of constitutive models is based on the numerical simulation of appropriate experiments. Within this context, the identification of material parameters is performed by an optimal fitting of numerical to experimental results. From a mathematical point of view this procedure represents an ill-posed inverse problem which can be solved using nonlinear optimization methods.

The development of efficient and stable algorithms for the numerical solution of multiphysics problems within the context of mixed finite element formulations has been intensified for the last years. Focusing on nearly incompressible hyperelastic as well as biphasic material behavior two essential well-investigated examples of mixed (u/p) formulations will be considered in this presentation. The corresponding models and tools for the direct as well as for the inverse problem are particularly directed towards the simulation of the mechanical behavior of soft biological tissues. Within this context, the identification of material parameters analyzing results of *in vitro* experiments under conditions close to the physiological state will be feasible.

The authors present an inverse method to identify the material parameters of the constitutive models mentioned above analyzing inhomogeneous displacement fields (cf. [1]). Within this context, the computation of the numerical values of the comparative quantities demands the solution of a complete time-consuming initial-boundary value problem. Therefore, deterministic (gradient based) optimization methods are preferred. Using a trust-region type method like the Levenberg-Marquardt approach, the

norm of the search step length will be limited by a properly chosen value. Least squares type objective functions are defined using local information (e.g. displacements) as well as global data (e.g. overall forces).

The numerical values of the comparative quantities are computed using a stable and efficient adaptive mixed (u/p) finite element approach. In case of nearly incompressible elasticity the material models under consideration are based on the multiplicative split of the deformation gradient into a deviatoric and a volumetric part (the so-called Flory split). For the simulation of biphasic material behavior the basic assumptions of the Theory of Porous Media are used (for an overview see e.g. [2]).

Within the context of deterministic optimization procedures the gradient of the objective function is required. Its calculation is performed within the scope of the sensitivity analysis. The authors study numerical and semianalytical approaches to define the gradient of the objective function. For both classes of mixed problems the semianalytical procedure is based on the implicit differentiation of the equilibrium condition, and the incompressibility condition (in case of nearly incompressible elasticity) or the volume balance of the mixture (in case of biphasic material behavior, cf. [3]) respectively. Starting point is the corresponding weak formulation of the governing equations mentioned above. Their consistent linearization in time results in the finite element formulation for the direct problem. Furthermore, the authors show that the implicit differentiation of the weak formulations with respect to a single material parameter after subsequent space discretization results in a mixed global system for the sensitivity analysis which is similar to that of the direct problem. The solution vector of this system consists of the parameter derivatives of the nodal displacements and the nodal pressure. The right-hand sides of the direct problem and the system for the sensitivity analysis differ, whereas the (tangential) stiffness matrices are equivalent for both problems and can be adopted from the solution of the direct problem.

The discussed direct as well as inverse problems are implemented into the author's in-house finite element code. This code combines thermodynamically consistent approximation of real material behavior with modern mathematical features including hierarchical error-controlled adaptive concepts and efficient iterative solvers. Within the context of optimization procedures elements of parameter estimation analyzing the corresponding covariance as well as correlation matrices are realized (cf. [4]). Some examples based on the reidentification of given material parameters considering the case of nearly incompressible elastic material behavior show the capabilities and limits of the presented method. The accuracy and the efficiency of the numerical and the semianalytical sensitivity analysis are analyzed in detail. Particularly in case of a comparatively large number of material parameters, the semianalytical procedure is much more efficient compared to the numerical approach.

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