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On Solving Discrete Topology Optimization Problems with Stress Constraints to Global Optimality

*** Mathias Stolpe[1], Nam Nguyen Canh[2] and Roman Stainko[3]**

[1] Dept. of Mathematics
Technical University
of Denmark (DTU)
DK – 2800 Kgs. Lyngby
Denmark
M.Stolpe@mat.dtu.dk

[2] Dept. of Mathematics
Technical University
of Denmark (DTU)
DK – 2800 Kgs. Lyngby
Denmark
N.N.Canh@mat.dtu.dk

[3] RISC Software GmbH
Softwarepark 35 (IT-Center)
A – 4232 Hagenberg
Austria
Roman.Stainko@gmail.com

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ABSTRACT

We consider structural topology optimization problems in which the design variables are chosen from a finite set of given values. The design variables can represent areas in truss structures, thicknesses in the two-dimensional case, and materials in the three dimensional situation. Our interest is in minimum weight problems with constraints on the global stiffness of the structure, i.e. the compliance, and on local stress properties, such as the von Mises stresses.

The main intention is to present the theoretical and practical aspects of a new convergent method for solving the considered class of problems to global optimality. The method is based on the concept of branch and cut ([2]) and will after a finite number of iterations (solved subproblems) correctly determine a global minimizer or deem the problem as infeasible.

We state the optimal design problem in a format based on a finite element discretization of the continuum problem. In this ground structure approach we have n finite elements in a two or three-dimensional design space with appropriate support conditions, see e.g. [1]. For a given vector $t \in \mathbb{B}^n$ of design variables the stiffness matrix of the structure in global coordinates is denoted by $K(t) \in \mathbb{R}^{d \times d}$. Here, d denotes the number of degrees of freedom. We throughout assume that the stiffness matrix $K(t)$ depends linearly on t, i.e.,

$$K(t) := \sum_{j=1}^{n} t_j K_j$$

where $t_j K_j$ is the scaled symmetric and positive semi-definite stiffness matrix of the j-th element. The discrete design variables can be interpreted as

$$t_j = \begin{cases} 1 & \text{if the } j\text{-th element contains material, and} \\ 0 & \text{otherwise.} \end{cases}$$

We consider M load conditions where the static loads are given by the vectors $f_1, \ldots, f_M \in \mathbb{R}^d \backslash \{0\}$ in global coordinates. The elastic equilibrium equations for the structure subjected to the static external load vector f_k is assumed to be given by $K(t)u_k = f_k$ for $k = 1, \ldots, M$, where $u_k \in \mathbb{R}^d$ denotes the nodal displacement vector corresponding to f_k. We consider the following minimum weight problem with constraints on the global stiffness and local stresses

$$
\text{(P)} \quad \begin{array}{lll}
\underset{t, u_1, \ldots, u_M}{\text{minimize}} & \displaystyle\sum_{j=1}^{n} t_j \rho_j & \\
\text{subject to} & K(t)u_k = f_k & \forall\, k \\
 & f_k^T u_k \leq \overline{\gamma}_k & \forall\, k \\
 & u_k^T W_j u_k \leq \overline{\sigma}^2 & \forall\, k, \forall\, j : t_j = 1 \\
 & t \in \{0,1\}^n. &
\end{array}
$$

The density ρ_j for the j-th element is assumed to be strictly positive. The compliances $f_k^T u_k$ for the different load cases are bounded by the given scalars $\overline{\gamma}_k > 0$ for $k = 1, \ldots, M$. The stress constrains are described by the symmetric and positive semi-definite matrices $W_j \in \mathbb{R}^{d \times d}$ and the stress bound $\overline{\sigma} > 0$. Notice that the stress constraints are only included in the formulation if the corresponding design variable is equal to one, i.e. if the corresponding element contains material.

In branch and cut methods a possibly very long sequence of relaxations, i.e. optimization problems which provide lower bounds on the objective function of the considered problem, are solved. These relaxations are ideally both tractable and good approximations of the considered discrete problem. One class of continuous relaxations of the discrete problem (P) is obtained by first (temporarily) removing some of the complicating constraints from the formulation, in this case the stress constraints, and then relaxing the constraints $t \in \{0,1\}^n$ to $t \in [0,1]^n$. The resulting continuous relaxation is a minimum weight problem with compliance constraints. This class of problems has been extensively studied and several convex and tractable reformulations of this relaxation are available, see [3] and references therein. We present new reformulations of this relaxation which are suitable for a practical implementation of a nonlinear branch and cut method. The relaxations and their reformulations also provide the foundation for several new heuristics, based on neighborhood search, designed to find good feasible designs and to improve on already found feasible designs.

The rate of convergence of the branch and cut method is closely related to the quality of the relaxations. Hence, we present an algorithm for generating valid linear inequalities, i.e. additional linear constraints, and cuts to strengthen the quality of the relaxations. This algorithm exploits the mathematical structure and the underlying mechanical assumptions of the problem, in particular the disjunctive nature of the constraints and variables.

The global optimization method is developed and implemented within the PLATO-N project and is used to solve benchmark examples which are used to validate other methods. The main aspects of the ongoing object oriented implementation of the method as well as numerical examples will be presented.

REFERENCES

[1] M.P. Bendsøe and O. Sigmund. *Topology Optimization – Theory, Methods and Applications*, Springer, 2003.

[2] R. Horst and H. Tuy, *Global Optimization: Deterministic Approaches*, Springer-Verlag, 1993.

[3] M. Stolpe and R. Stainko and M. Kočvara. "Lower bounding problems for stress constrained discrete structural topology optimization problems". *EU project FP–6 STREP 30717 PLATO–N (Aeronautics), PLATO–N Public Report PU–R–5–2007*, 2007.